Rigorous Approach to Coupling Matrix Synthesis Problem With Arbitrary Topology

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***Abstract*—This project discusses a procedure of finding every coupling matrices complying with a given target topology using the similarity transformation method. This method is applicable regardless of the number of solutions determined by the target topology. When the target topology has an infinite number of solutions, this method yields a parameterization of every possible solutions with free variables. This procedure can be regarded as a generalization of the conventional 'pivot'-based method.**

***Index Terms*—Coupling matrix, filter synthesis, multiple solutions, similarity transformation, topology reconfiguration**

I. Introduction

In microwave filter engineering, finding different physical structures with the same frequency response is often desirable, as it increases the design flexibility of physical implementation. Designing a physical filter structure requires a corresponding theoretical prototype circuit, often represented as a coupling matrix. This square matrix, $\mathbf{M}$, describes the mutual couplings within a given microwave filter circuit. The topology of the prototype circuit reflects the architecture of the physical filter and is directly encoded into the coupling matrix. Therefore, synthesizing a coupling matrix with a topology that matches the designer's physical filter structure is crucial.

Given a frequency response, the number of coupling matrices that yields the response may vary depending on the topology. In other words, if an engineer were to design a microwave filter satisfying the given specification, there may be a room for changing the physical parameter values depending on the topology of the structure. Some topologies have a finite number of solution (i.e. a finite number of choices for physical parameters) while others have an infinite number of solutions. The former topologies are called *non-redundant*, while the latter topologies are called *redundant* [1].

Meanwhile, it is known that applying specific *similarity transformation* to this coupling matrix $\mathbf{M}$ yields another coupling matrix $\mathbf{M}'$ with the same frequency response. Similarity-transforming a matrix is essentially changing the basis of representation, which is done by multiplying an orthonormal matrix $\mathbf{R}$ to both sides of $\mathbf{M}$ as follows; $\mathbf{R}^{\mathrm{T}} \cdot \mathbf{M} \cdot \mathbf{R} = \mathbf{M}'$. This mathematical technique has been widely used to find solutions for different topologies, both by optimization and rigorous analysis methods [2]−[3].

However, current methods have two main downside; 1) they do not provide insights into the mathematical structure of the problem, and 2) they find difficulties when applied for redundant topologies, especially when the method is based on numerical optimization. To address these issues, this work proposes a rigorous synthesis procedure of arbitrary-topology coupling matrices, based on 'geometric interpretation' of the similarity transformation method. This method provides a mathematical approach for finding *all* solutions of both non-redundant and redundant topology.

This report presents the summary of the work supported in part by the MTT-S Scholarship program and the details can be found in [4].

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II. Synthesis Procedure

Our synthesis approach begins with following two observations. 1) Given a matrix $\mathbf{M}$ in one topology, to find a matrix $\mathbf{M}'$ in another topology is to find an orthonormal matrix $\mathbf{R}$, where its column vectors, r_i's, are orthogonal to some transformed column vectors, $\mathbf{M}r_j$'s. 2) An arbitrary orthonormal matrix $\mathbf{R}$ can be fully parameterized by expressing r_i's in n-spherical coordiantes with change-of-basis transformations.

A. Observation 1)

It is known that given a coupling matrix $\mathbf{M}$, any other coupling matrix $\mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R} = \mathbf{M}'$ yields the same frequency response as $\mathbf{M}$, where

$$\mathbf{R}^{\mathrm{T}}\mathbf{R} = \mathbf{I}, \quad r_0 = \pm e_0, \quad r_{n+1} = \pm e_{n+1}. \tag{1}$$

Thus, given a coupling matrix $\mathbf{M}$, to find coupling matrices $\mathbf{M}'$'s in the desired topology is to find orthonormal matrices $\mathbf{R}$ where $\mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R} = \mathbf{M}'$ satisfies the target-topology conditions. Specifically, if the target topology demands that there be no coupling between the ith and the jth nodes, $\mathbf{M}'_{i,j}$ should be zero. Then, from the rule of matrix multiplications, we have

$$\mathbf{M}'_{i,j} = 0 \iff (\mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R})_{i,j} = 0 \iff {r_i}^{\mathrm{T}}\mathbf{M}r_j = 0. \tag{2}$$

The last equation in (2) indicates that r_i must be orthogonal to $\mathbf{M}r_j$, which is our first observation.

B. Observation 2)

For a redundant target topology, there is an infinite number of solutions, thus infinitely many $\mathbf{R}$'s that yield $\mathbf{M}'s$ in the target topology. Thus, we need a systematic way to parameterize such an infinite number of orthonormal matrices, $\mathbf{R}$'s.

An orthonormal matrix $\mathbf{R}$ is an $n \times n$ matrix where 1) each column vector, r_i, is a unit vector and 2) each column vectors are orthogonal to each other. From the first condition, we can

	0	1	2	3	4	5	6	7
0	0	1.0018	0	0	0	0	0	0
1	1.0018	0.0242	0.8429	0	0	0	0	0
2	0	0.8429	0.0307	0.6090	0	-0.0695	0	0
3	0	0	0.6090	-0.0332	-0.4028	-0.3767	0	0
4	0	0	0	-0.4028	-0.6673	0.4785	0	0
5	0	0	-0.0695	-0.3767	0.4785	0.0307	0.8429	0
6	0	0	0	0	0	0.8429	0.0242	1.0018
7	0	0	0	0	0	0	1.0018	0

(a)

	0	1	2	3	4	5	6	7
0	0	1.0018	0	0	0	0	0	0
1	1.0018	0.0242	0.4554	-0.7093	0	0	0	0
2	0	0.4554	-0.4635	-0.3173	0.3652	0	0	0
3	0	-0.7093	-0.3173	-0.1731	0.3367	-0.7212	0	0
4	0	0	0.3652	0.3367	0.0510	-0.3744	0.8426	0
5	0	0	0	-0.7212	-0.3744	-0.0535	0.0229	0
6	0	0	0	0	0.8426	0.0229	0.0242	1.0018
7	0	0	0	0	0	0	1.0018	0

(b)

	0	1	2	3	4	5	6	7
0	0	1.0018	0	0	0	0	0	0
1	1.0018	0.0242	0.3650	-0.7598	0	0	0	0
2	0	0.3650	-0.0863	-0.5629	-0.5546	0	0	0
3	0	-0.7598	-0.5629	-0.4831	0.1152	0.0269	0	0
4	0	0	-0.5546	0.1152	0.0611	0.6030	0.1115	0
5	0	0	0	0.0269	0.6030	-0.1308	0.8355	0
6	0	0	0	0	0.1115	0.8355	0.0242	1.0018
7	0	0	0	0	0	0	1.0018	0

(c)

Fig. 1. (a) Initial matrix **M**. (b) Matrix **M′** with one set of free variables. (c) Matrix **M′** with another set of free variables.

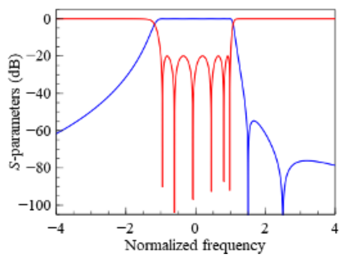

Fig. 2. A 6th order frequency response corresponding to the three coupling matrices in Fig. 1(a), (b) and (c). It has 20-dB equi-ripple return loss and two transmission zeros at $\Omega = 1.5$ and 2.5 (Ω: normalized frequency).

parameterize one of its column vectors, namely r_1, using n-spherical coordinates as follows.

$$r_1 = \left[\prod_{i=1}^{n-1} C_i^1 \quad S_{n-1}^1 \prod_{i=1}^{n-2} C_i^1 \quad S_{n-2}^1 \prod_{i=1}^{n-3} C_i^1 \quad \ldots \quad S_2^1 C_1^1 \quad S_1^1 \right]^{\mathrm{T}}, \tag{3}$$

where C_i^k and S_i^k denote $\cos(\theta_{k,i})$ and $\sin(\theta_{k,i})$, respectively. The index k is to distinguish free variables θ's used for r_1 from those used for other r_i's.

In order to parameterize the next column vector r_2, we incorporate a rotation matrix $\mathbf{P}_1$, where $\mathbf{P}_1 r_1 = e_1$. Essentially, $\mathbf{P}_1$ is a change-of-basis transformation, where r_1 is one of the basis vectors in the new basis. Transformation $\mathbf{P}_1$, being an orthonormal transformation, preserves inner products between any two vectors. Thus, $r_1 \perp r_2$ is equivalent to $\mathbf{P}_1 r_1 \perp \mathbf{P}_1 r_2$. Then, arbitrary r_2 can be parameterized using $(n-1)$-spherical coordinates in this changed basis as follows.

$$\mathbf{P}_1 r_2 = \left[0 \quad \prod_{i=1}^{n-2} C_i^2 \quad S_{n-2}^2 \prod_{i=1}^{n-3} C_i^2 \quad \ldots \quad S_2^2 C_1^2 \quad S_1^2 \right]^{\mathrm{T}}. \tag{4}$$

Then r_2 can be obtained by left-multiplying $\mathbf{P}_1^{\mathrm{T}}$ to (4). One can repeat this procedure untill all r_i's are determined.

Notice that to parameterize arbitrary $\mathbf{R}$ is to find r_i's that satisfy some orthogonality relations between them; $r_i \perp r_j$ if $i \neq j$. Then, with observation 1), to find a matrix $\mathbf{M}'$ in target topology is to find r_i's that satisfy additional orthogonality relations; $r_i \perp \mathbf{M} r_j$ if $\mathbf{M}'_{i,j}$ should be 0. In other words, one can handle the coupling matrix synthesis problem as a generalized version of parameterizing arbitrary orthonormal matrix, with more orthogonality relations to be satisfied. With this procedure, one can find all $\mathbf{M}'$'s satisfying target-topology conditions. If the target topology is redundant, the solutions are parameterized with possibly multiple free variables, θ's.

III. Results

We have confirmed that many non-redundant and redundant topologies can be solved with this synthesis approach. Numerical results of one such redundant-topology example is given throughout Figs. 1 and 2. Matrix values of **M** and **M′**'s are given in Fig. 1 and the corresponding frequency response is given in Fig. 2. The blank entires of each matrix in Fig. 1 indicate that the corresponding topology demands them to be zero. The colored entries indicate that the corresponding topology allows them to be nonzero. Fig. 1(a) shows the initial topology and numerical values of **M** that yield the response in Fig. 2. Similarly, Fig. 1(b) and (c) show the target topology and numerical values of two **M**'s, out of infinitely many, that yield the response in Fig. 2. The initial topology is a well-known topology, where one can readily obtain the corresponding **M** values given a frequency response. We parameterized every possible solutions for the target topology, and calculated two solutions in Fig. 1(b) and (c) with different free-variable values using the initial matrix **M** in Fig. 1(a).

IV. Conclusion

This report presents the summary of work in [4], partially supported by MTT-S scholarship program. An analytic approach for synthesizing coupling matrices with arbitrary topology is investigated. The proposed method is capable of finding every possible coupling matrices for both non-redundant and redundant topologies. It is confirmed that an infinite number of solutions of a redundant topology can be parameterized with the proposed procedure.

V. Acknowledgement

I am greatly honored to receive the MTT-S Undergraduate Scholarship. The scholarship program has granted me an opportunity to explore the theories of RF engineering, especially the mathematics of microwave filter synthesis. I found the related mathematical theories very interesting, and decided to further study them by pursuing an M.S. degree at Korea University. I am looking forward to attend other MTT-S conferences, if any opportunity is provided.

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