Ferromagnetic and Ferroelectric Thin Films Enabled Nonlinear Circuits for Environment Adaptive Position Independent Wireless Power Transfer Technologies

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Abstract—This ongoing project aims to reduce the positional sensitivity of wireless power transfer (WPT) circuits using ferromagnetic and ferroelectric thin-film technologies. WPT circuits are known to be susceptible to misalignment in the transmitting and receiving elements or environmental effects. Recent research shows that it is possible to compensate for these problems using nonlinear components. In this work, multiple methods of simulating a Duffing resonator are compared to study the effects of nonlinearity. Additionally, an experimental nonlinear resonator circuit has been constructed. While certain characteristics of the Duffing resonator are observed in the real circuit, simulations of the real circuit have some disagreements with the actual results. Ferromagnetic and ferroelectric thin-film engineered substrates have also been developed, and their usability as electrically tunable passive components is demonstrated. In future work, the engineered substrates can be used to create custom nonlinear components for use in positionindependent WPT circuits.

Index Terms—Engineered substrate, ferroelectric thin films, ferromagnetic thin films, nonlinear resonator, wireless power transfer (WPT)

I. INTRODUCTION

WIRELESS POWER TRANSFER (WPT) is an emerging technology with a significant impact in many applications where wired powering is either impractical or inconvenient. A desirable feature of WPT systems is consistent power delivery for different distances and angular alignments between the transmitting and receiving coils. However, in a linear WPT system with constant frequency, power transfer efficiency (PTE) is severely reduced for non-optimal positioning [1]. Recently, it has been demonstrated in [2-4] that nonlinear capacitors can compensate for changes in position and maintain high PTE for a range of distances and angular alignments between the coils. The nonlinear circuit theory presented in [2-4] is based on the Duffing equation, which describes a class of resonators with a third-order nonlinearity. Tunable ferromagnetic and ferroelectric thin film technologies can be used to design components with the desired nonlinearity.

This work presents the preliminary results of simulation and experiments with nonlinear resonators. In addition, the tunability of an engineered substrate with a ferromagnetic thin film is demonstrated.

II. DUFFING RESONATOR

The Duffing equation is a well-studied nonlinear differential equation that describes a forced oscillator with a third-order nonlinear restoring force [5]. It can be expressed as

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + \alpha x + \beta x^3 = \Gamma \cos \omega t, \tag{1}$$

where x(t) is the response variable, t is time, ω is frequency, and k, α , β , and Γ are coefficients that depend on the system.

An RLC resonator with a nonlinear capacitor can be described by (1) if a third-order approximation of the capacitor's voltage-charge relationship is used as in [2-4], i.e. $v_c = \frac{1}{c}q + \frac{1}{c'}q^3$, (2) where *v* is voltage *q* is charge *C* is the linear component of

where v_c is voltage, q is charge, C is the linear component of capacitance, and C' is the nonlinear component.

Thus, for a resonator with resistance R, inductance L, and voltage source amplitude V_s , the Duffing equation for the circuit can be written as

$$L\frac{d^{2}q}{dt^{2}} + R\frac{dq}{dt} + \frac{1}{c}q + \frac{1}{c'}q^{3} = V_{s}\cos\omega t.$$
 (3)

In [2] and [3], a frequency-domain method is used to solve (3), whereas the method of multiple scales is used in [4], based on [6]. In this work, both methods have been used to solve the equation in MATLAB, and the results have been compared to each other as well as the results from the commercial circuit simulator Keysight Advanced Design System (ADS).

In the simulations, $V_s = 50$ V, L = 1 mH, $R = 15 \Omega$, C = 1 nF, and C' = 1e-19 C³/V (coulombs cubed per volt). Frequency was swept from 120 kHz to 200 kHz. Plots of capacitor charge for the methods used in MATLAB are shown in Fig. 1. Both methods are in strong agreement, with a bistable region appearing between 160 kHz and 170 kHz and peak charge amplitudes within 5% of each other.

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Fig. 1. Comparison of Duffing equation solutions for frequency-domain and multiple-scale methods. The frequency responses are nearly identical.

The same component values were used to simulate the circuit in ADS using the built-in nonlinear capacitor component. The same voltage-charge relationship was used. Harmonic balance was chosen to solve the circuit because it allows for frequencydomain simulation of nonlinear systems. Additionally, the hysteretic effect of the Duffing resonator can be demonstrated by altering the direction of the frequency sweep. This allows for



Fig. 2. Forward and reverse frequency sweep results for the Duffing resonator from ADS. The peak amplitude is close to the MATLAB results, and a small bistable region is present. The inset shows a close-up of the bistable region. both the lower and upper branch to be captured. The results are shown in Fig. 2. The bistable region is small compared to the results in MATLAB, but the peak charge amplitude and the frequency at which it occurs are similar. This validates the solution methods used in [2-4].

The experimental circuit involved a 760 μ H inductor, and 4 anti-series pairs of Skyworks SMV1249 varactor diodes were used for the capacitance. The input signal was supplied by an HP 8657B RF source with an output of -5 dBm. The frequency of the input signal was swept between 500 kHz and 1 MHz in forward and reverse order to capture the bistable region. As predicted by the simulations, the hysteretic effect could be



Fig. 3. Forward and reverse frequency sweep results for the experimental nonlinear resonator. Hysteresis is demonstrated as multiple stable states appear for the same conditions, depending on which direction the frequency is swept. observed, with different states occurring the same frequency depending on the initial conditions. However, simulations with the real circuit's values tended to disagree with the actual results, likely due to the increased nonlinearity.

III. TUNABLE ENGINEERED SUBSTRATE

A tunable phase shifter and tunable bandpass filter were constructed as simple demonstrations of the engineered substrate. A variable dc current was passed through the substrate in each system and could be varied to alter their characteristics. Fig. 4 shows the measured S_{12} for the phase shifter and the bandpass filter for varying currents. More details can be found in the full paper on the ferroelectric and ferromagnetic materials in [8].



Fig. 4. Measured S_{12} for a phase shifter (left) and bandpass filter (right) for a range of dc currents passed through the engineered substrate. The results demonstrate the tunability of the substrate.

IV. CONCLUSION

Going forward, we will continue to study nonlinear dynamics in order to determine the ideal nonlinearity for a WPT circuit. Further development of tunable engineered substrates will also enable their use in resonator or WPT circuits with specifically designed characteristics.

Receiving this scholarship has inspired the student, David West, to delve deeper into research and strongly consider pursuing a graduate degree in RF engineering. He enjoyed the opportunity to attend the 2021 International Microwave Symposium in Atlanta and was deeply impressed by the breadth of topics covered there by experts from around the world.

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