

On Stripline Y-Circulation at UHF*

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Summary—The simplified boundary-value problem of the circular strip-line Y-circulator is stated and, following experimental results, is solved approximately. The circulation parameters are calculated and discussed. The frequency characteristics are evaluated and a general method for broadbanding the device is presented. From the calculated and measured field distribution, an explanatory description of the circulation mechanism is given. Finally, other possible solutions and the features for use at UHF are discussed.

INTRODUCTION

THE STRIPLINE Y-circulator (Fig. 1) polarized above the resonance field has been found to have useful properties in the UHF region and, consequently, has received much attention.¹⁻⁵ In its basic form, it is a member of a larger family of nonreciprocal devices, the junction circulators. The general properties including the existence of and the tuning procedure for the symmetrical junction circulators have been studied by Auld⁶ through the consideration of their scattering matrices and associated quantities. For the stripline Y-circulator in particular, the same has been done by Milano,¹ *et al.*

However, as far as the present author is aware, no consistent theory is yet available about the intrinsic circulator mechanism in terms of the electromagnetic field. The same can be said about the frequency characteristics of these types of circulators. It is true that several attempts have been made to explain the junction circulator in terms of Faraday rotation,^{7,8} asymmetrical diffraction⁹ or field displacement⁵ but, although these efforts have confirmed, more or less, the intuitive thinking about the phenomenon, they have not led to a consistent and manageable theory. In a recent paper,

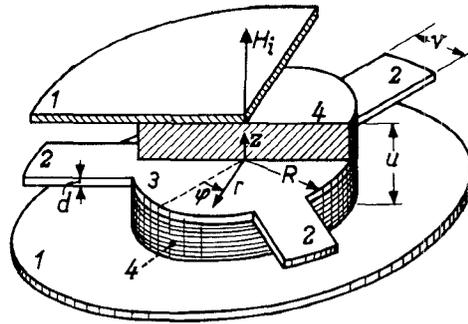


Fig. 1—The circulator configuration. (1) Outer conductors. (2) Inner conductors. (3) Center conductor. (4) Ferrite disks.

Skomal¹⁰ explained the junction circulator in terms of two contrarotating surface waves. He evaluated some of the design parameters. However, some of his results, *e.g.*, the dependence of applied field on magnetization, do not agree with the findings of the present paper. This may be due to the fact that some of the assumptions made by Skomal are not always tenable. In particular, the circumferential distance between adjacent ports is not equal to the effective wavelength in all cases¹¹ (see below).

The investigation by Bosma,⁵ which dealt with the 3-port stripline Y-circulator as this paper does, looked quite promising, but it has turned out that some of its results were not in agreement with experiment. Although the problem was stated correctly, the attempt, as is shown below, failed because a practically wrong (although mathematically nearly possible) solution was traced.

In this paper, the same problem is restated and, following the results of experiment,¹² an approximation of the problem is proposed. Then, again led by experimental results, a supposition for a practical solution is made. This solution is tested in the circulator equations and is shown to be consistent with other theoretical considerations. It also agrees well with all experimental results available.

From the solution, first order approximation, of the frequency dependence of the circulator characteristics are obtained. A general method of broadbanding the circulator is proposed. These results are also confirmed by experiment.

¹⁰ E. N. Skomal, "Theory of operation of a 3-port Y-junction ferrite circulator," *IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. 11, pp. 117-123; March, 1963.

¹¹ J. B. Davies and P. Cohen, "Theoretical Design of Symmetrical Junction Stripline Circulators," *IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. MTT-11, pp. 506-512; November, 1963.

¹² Since this paper is dealing with the theory of the stripline circulator, the experiments and their results are not described in detail. They will probably be published elsewhere.

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¹ U. Milano, J. Saunders and L. Davis, "A Y-junction stripline circulator," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. 8, pp. 346-351; May, 1960.

² S. Yoshida, "J-Band stripline Y circulator," *PROC. IRE*, vol. 48, p. 1664; September, 1960.

³ L. Freiberg, "Lightweight Y-junction stripline circulator," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. 8, p. 672; November, 1960.

⁴ G. V. Buehler and A. F. Eikenberg, "Stripline Y-circulator for the 100 to 400 Mc region," *PROC. IRE*, vol. 49, p. 518; February, 1961.

⁵ H. Bosma, "On the principle of stripline circulation," *Proc. IEE*, vol. 109, pt. B, suppl. no. 21, pp. 137-146; January, 1962.

⁶ B. A. Auld, "The synthesis of symmetrical waveguide circulators," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. 7, pp. 238-247; April, 1959.

⁷ P. J. Allen, "The turnstile circulator," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. 4, pp. 223-228; October, 1956.

⁸ C. Bownes, "Discussion on microwave ferrites II," *PROC. IEEE*, vol. 109, pt. B, suppl. no. 21, p. 163; January, 1962.

⁹ V. G. Feoktistov, "Diffraction model of Y-circulator," *Radio-tekhnika i Elektronika*, vol. 7, pp. 1763-1768; October, 1962.

Furthermore, it will be found that circulator action is based on a slight distortion of the degeneracy of two first order resonances of the "isotropic" disk configuration caused by a small amount of gyrotropy of the disk material. Under the amount of gyrotropy, we understand the ratio κ/μ , where κ and μ are the elements of the Polder tensor [cf., (12)]. The small amount of gyrotropy introduces a small asymmetric distortion of the isotropic field configuration. This asymmetry is necessary for a device to be nonreciprocal.

The smallness of the gyrotropy is an essential feature. As a consequence, the internal polarizing static magnetic field H_i is much larger than the internal resonance field¹³ H_0 . In most cases, the former is more than four times as large as the latter. In the opinion of the author, this is the main reason why the 3-port stripline Y-circulator is so effective in the UHF region.

PROPERTIES OF THE SCATTERING MATRIX

Before the program outlined above is started, some remarks on the scattering matrix of a lossless, cyclic-symmetric 3-port may be made. The scattering matrix S may be written in the form

$$S = \begin{pmatrix} \alpha & \gamma & \beta \\ \beta & \alpha & \gamma \\ \gamma & \beta & \alpha \end{pmatrix}. \quad (1)$$

Carlin¹⁴ has shown that a matched, lossless 3-port is a circulator and Thaxter and Heller¹⁵ have pointed out that, if the reflection-coefficient α is small $|\alpha| \ll 1$, the equations

$$|\gamma| = |\alpha|, \quad |\beta| = 1 - 2|\alpha|^2 \quad (2)$$

hold approximately. For a circulator which rotates in the opposite sense, γ is interchanged with β . An elegant description of these properties has been given by Butterweck.¹⁶

The elements of S satisfy the equations

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1 \quad (3)$$

$$\alpha\beta^* + \beta\gamma^* + \gamma\alpha^* = 0. \quad (4)$$

By virtue of (3), $|\beta|$ can be expressed in terms of $|\alpha|$ and $|\gamma|$. The three terms of (4) can be interpreted as three vectors which span a triangle. Then, inequality relations like

$$|\beta| |\gamma| \leq |\alpha| (|\beta| + |\gamma|) \quad (5)$$

¹³ For reasons of simplicity, we presume that H_0 is given by $H_0 = \omega/\gamma$, where $\gamma = 2\pi \cdot 2.8 \text{ McOe}$, although in the configuration in question the ferromagnetic resonance occurs at an internal bias field which can be solved from $\omega = \gamma \sqrt{H(H + 4\pi M)}$, where $4\pi M$ is the magnetization of the ferrite. H_0 is used as a reference quantity only.

¹⁴ H. J. Carlin, "Principles of gyrator networks," *Proc. Symp. on Modern Advances in Microwave Techniques*, Polytechnical Institute of Brooklyn, N. Y., p. 175-204; November, 1954.

¹⁵ J. B. Thaxter and G. S. Heller, "Circulators at 70 and 140 kMc," *Proc. IRE*, vol. 48, pp. 110-111; January, 1960.

¹⁶ H. J. Butterweck, "Der Y-zirkulator," *Archiv der Elektrischen Uebertragung*, vol. 17, p. 163-176; April, 1963.

are valid. From these considerations Butterweck deduced that in the $|\alpha|, |\gamma|$ diagram the possible 3-ports are restricted to a region which is bounded by three ellipses. In Fig. 2 this region is indicated by the shaded area. The origin of the diagram ($|\alpha|, |\gamma| = (0, 0)$) represents an ideal clockwise rotating circulator. Starting from this point, it is easy to show that, for $|\alpha| \ll 1$, $|\gamma|$ is given by the series

$$|\gamma| = |\alpha| + \theta |\alpha|^2 + \dots, \quad -1 \leq \theta \leq 1. \quad (6)$$

Then, by virtue of (3), $|\beta|^2$ is given by the series

$$|\beta|^2 = 1 - 2|\alpha|^2 - 2\theta |\alpha|^3 \dots \quad (7)$$

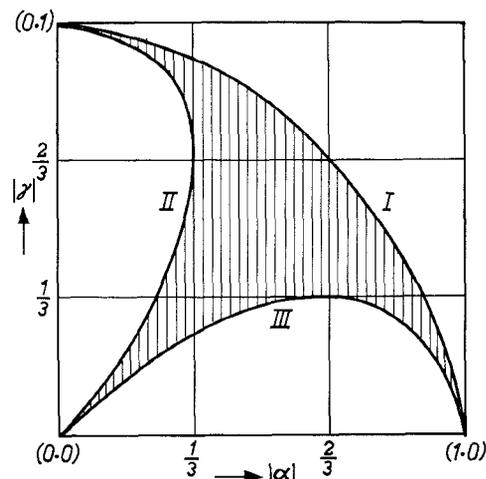


Fig. 2—The closed region of possible, lossless and cyclic-symmetric 3-ports, (Butterweck¹⁶).

$$\begin{aligned} \text{(I)} \quad & |\alpha|^2 + |\alpha| |\gamma| + |\gamma|^2 - |\alpha| - |\gamma| = 0. \\ \text{(II)} \quad & |\alpha|^2 - |\alpha| |\gamma| + |\gamma|^2 + |\alpha| - |\gamma| = 0. \\ \text{(III)} \quad & |\alpha|^2 - |\alpha| |\gamma| + |\gamma|^2 - |\alpha| + |\gamma| = 0. \end{aligned}$$

THE 3-PORT STRIPLINE Y-CIRCULATOR

In Fig. 1, a picture of the circulator is given. The cylindrical coordinate system, (r, ϕ, z) , used further on, has been drawn into it and several of the important dimensions are presented.

The two ferrite disks are placed between the two earth plates (the outer conductors) and the center conductor. They are magnetized perpendicularly to the plane of the conductors by a static magnetic field. The value of the internal polarizing field (in the ferrite) is denoted by H_i .

In the elementary case, the ferrite disks and the center conductor both have circular form and all three have the same radius R . In practice this need not be so. The disks and the center conductor may have different sizes and forms. Other forms, such as triangles or clover⁴ leaves, can be used if only a 3-fold rotation symmetry is present; this, in order to maintain the 3-fold cyclic symmetry of the circulator. Moreover, dielectric inserts may be present. For mathematical simplicity only the elementary configuration is investigated in this paper.

At equal distances around the edge of the center conductor, three inner conductors of stripline waveguides

are connected. The centers of these connections are taken at ϕ -values $-\pi/3$, $\pi/3$ and π for the input, output and decoupled line, respectively. If v is the width of the stripline, a stripline width angle Ψ is defined by (see Fig. 3)

$$\Psi = \sin^{-1}(v/2R). \quad (8)$$

The thickness d of the inner and center conductors is assumed to be zero. The distance between the two outer conductors is u , twice the thickness of one ferrite disk.

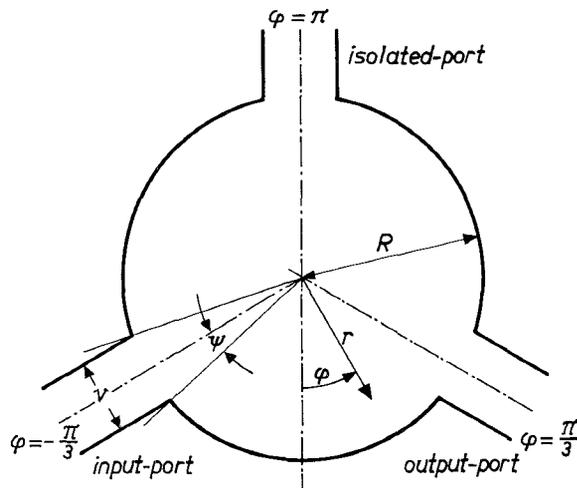


Fig. 3—The configuration of the center conductor.

FIELD CONFIGURATION IN AND CHARACTERISTIC IMPEDANCE OF THE STRIPLINES

In the cross section of the striplines, the field intensities are supposed not to vary over the width of the inner conductors and to be zero outside these conductors. The stray fields at the edges are neglected. However, especially in the neighborhood of the connections to the center conductor, rather large differences from the assumed ideal field configuration can be expected to exist. This supposition made about the fields in the stripline is clearly an approximation only.

The striplines carry TEM modes only. Hence, the electric field intensities are perpendicular to the conductors (in the z -direction) and the magnetic field intensities are parallel to them and perpendicular to the directions of propagation (in the ϕ -direction). Furthermore, the fields on the two sides of the inner conductors are 180° out of phase.

The characteristic impedance of the striplines is not important, because of the method of analysis presented. For convenience of matching to the other elements of the experimental setups, the characteristic impedances are made equal to 50Ω in all practical cases. This is done by the proper choice¹⁷ of u with respect to v (and d).

The ratio of the electric and magnetic field intensities of a traveling wave, the intrinsic wave impedance ζ , is

of much more importance for the analysis. In the simple model of the circulator, no dielectric materials other than air are placed between the conductors of the striplines. Hence, ζ is given by

$$\zeta = \sqrt{\mu_0/\epsilon_0} = 120\pi\Omega. \quad (9)$$

MAGNETIC FIELD INTENSITY AT THE EDGE OF A DISK

As is the case with the fields in the striplines, the electric as well as the magnetic field intensities on either side of the center conductor are equal and oppositely directed at any instant. Hence, the field problem need be solved only for one disk in association with the fields on the appropriate sides of the inner conductors of the striplines.

Furthermore, it is supposed that the field in a disk does not depend on the z -coordinate, so that the problem can be reduced to two of three dimensions.

Except at the connections of the striplines, no radial current can flow from the edge of the center conductor. From boundary equations, it is known that, at the connections, the tangential component of the magnetic field intensity in the ferrite at its edge $H_\phi(R, \phi)$ is equal to the magnetic field intensity in the corresponding stripline at the same place. Consequently, $H_\phi(R, \phi)$ is constant over the stripline widths and it is zero elsewhere.¹⁸ This can be expressed by

$$H_\phi(R, \phi) = \begin{cases} \text{a) } -\pi/3 - \Psi < \phi < -\pi/3 + \Psi, \\ \text{b) } \pi/3 - \Psi < \phi < \pi/3 + \Psi, \\ \text{c) } \pi - \Psi < \phi < \pi + \Psi, \\ \text{d) elsewhere.} \end{cases} \quad (10)$$

The fields at the different connections need not be in phase, so that in principle *a*, *b* and *c* are complex numbers.

The investigation in this paper will be restricted to values of v which are not too large in comparison with R ; more precisely

$$v < R. \quad (11)$$

The smaller the ratio v/R is [cf., (8)], the more accurate is the assumed distribution of $H_\phi(R, \phi)$, but the larger is the disturbing influence of the stray fields of the striplines. Hence, v must not be very small.

ELECTRIC FIELD INTENSITY IN THE DISK

The electric field intensity in the disk is assumed to have a z -component only. It is denoted $E_z(r, \phi)$. The specific permittivity of the ferrite is denoted by ϵ and the

¹⁸ Hereafter, it is shown that the electric field intensity at the edge of the disk is not equal to zero. Thus, an electric stray field which has a radial component is present. Consequently, the supposition $H_\phi(R, \phi) = 0$ outside the connections is not exact. However, from a theoretical estimation and also from experiments, it appeared that the resultant inaccuracy is not larger than those of the other approximate suppositions.

¹⁷ R. W. Peters *et al.*, "Handbook of Triplate Microwave Components," Sanders Associates, Inc., Nashua, N. H. 1956.

specific gyrotropic permeability by $\|\mu\|$. The latter has a tensor character and is given by

$$\|\mu\| = \begin{bmatrix} \mu & -i\kappa & 0 \\ i\kappa & \mu & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (12)$$

An effective specific permeability μ_{eff} can be introduced by

$$\mu_{\text{eff}} = (\mu^2 - \kappa^2)/\mu \quad (13)$$

and an intrinsic wave number k by

$$k^2 = \omega^2 \mu_0 \epsilon_0 \mu_{\text{eff}} \epsilon. \quad (14)$$

With these definitions made, it is shown by Bosma⁵ that $E_z(r, \phi)$ satisfies the homogeneous Helmholtz equation

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k^2 \right] E_z(r, \phi) = 0 \quad (15)$$

and that the tangential and radial components of the magnetic field intensity in the disk are related to $E_z(r, \phi)$ by, respectively,

$$H_\phi(r, \phi) = i \left[\frac{\partial E_r}{\partial r} + i \frac{\kappa}{\mu} \frac{1}{r} \frac{\partial E_r}{\partial \phi} \right] / \omega \mu_0 \mu_{\text{eff}} \quad (16)$$

$$H_r(r, \phi) = -i \left[\frac{1}{r} \frac{\partial E_r}{\partial \phi} - i \frac{\kappa}{\mu} \frac{\partial E_r}{\partial r} \right] / \omega \mu_0 \mu_{\text{eff}}. \quad (17)$$

If $H_\phi(r, \phi)$ is known at the edge [cf., (10)], then, by virtue of (16), $E_z(r, \phi)$ is determined by oblique boundary conditions. Then, a Green's function $G(r, \phi; r', \phi')$ can be introduced¹⁹ such that

$$E_z(r, \phi) = \int_{-\pi}^{\pi} G(r, \phi; R, \phi') H_\phi(R, \phi') d\phi'. \quad (18)$$

In most cases, one is interested in $E_z(r, \phi)$ at the edge only and, therefore, in order to simplify the equations, one may write

$$G(R, \phi; R, \phi') = G(\phi; \phi'). \quad (19)$$

From the distribution $H_\phi(R, \phi)$ given by (10), $E_z(R, \phi)$ can be evaluated at the centers of the connections.²⁰ For small values of Ψ , the following approximate equations result:

$$E_z(R, -\pi/3) = A = 2\Psi [G(-\pi/3; -\pi/3)a + G(-\pi/3; \pi/3)b + G(-\pi/3, \pi)c] \quad (20)$$

$$E_z(R, \pi/3) = B = 2\Psi [G(\pi/3; -\pi/3)a + G(\pi/3; \pi/3)b + G(\pi/3, \pi)c] \quad (21)$$

¹⁹ D. van Dantzig and H. A. Lauwerier, "The North Sea problem," *Proc. Koninklijke Nederlandse Akademie van Wetenschappen*, series A (mathematical sciences), vol. LXIII, p. 170-180; 1960.

²⁰ Instead of evaluating $E_z(R, \phi)$ at the centers of the striplines, the average values of $E_z(R, \phi)$ over the stripline widths could have been determined. This more complicated method has been carried out by Bosma,⁵ but its results are not much more accurate.

$$E_z(R, \pi) = C = 2\Psi [G(\pi; -\pi/3)a + G(\pi; \pi/3)b + G(\pi; \pi)c]. \quad (22)$$

Since E_z is parallel to the cylindrical air-ferrite interface, it is the same on both sides of the edge. Therefore, A , B and C are also the electrical field intensities in the corresponding striplines.

Eqs. (20)–(22) express the relations between the RF-em fields in the striplines (the incident and reflected waves) in terms of the properties of the field pattern in the disk. Hence, the scattering matrix elements of the 3-port circulator can be expressed in terms of $G(\phi; \phi')$.

PROPERTIES OF $G(\phi; \phi')$

The Green's function can be interpreted as a transfer impedance function for the fields in the disk configuration. Before it is evaluated for the particular case of this paper, the conditions imposed upon it by general circulator characteristics are investigated.

A. Losslessness

If the field configuration at the boundary of the disk is known, by Poynting's theorem the average power which is radiated into or out of the disk can be calculated. For a lossless 3-port this must be zero. With the complex Poynting vector \mathbf{P} ,

$$\mathbf{P} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^*, \quad (23)$$

in which the asterisk denotes the complex conjugate, this is expressed by

$$\text{Re} \left[\int_S \mathbf{P} \cdot \mathbf{n} dS \right] = 0, \quad (24)$$

where \mathbf{n} is normal to the closed ferrite surface S . As the normal component of \mathbf{P} is nonzero only on the cylindrical part of the ferrite surface, by virtue of (18), (19), (23) and (24) it is easy to show that losslessness of the disk implies that the equation

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} H_\phi^*(R, \phi) [G(\phi; \phi') + G^*(\phi'; \phi)] \cdot H_\phi(R, \phi') d\phi d\phi' = 0 \quad (25)$$

must be satisfied. Since this must be valid for all distributions $H_\phi(R, \phi)$, the condition for losslessness is

$$G(\phi'; \phi) = -G^*(\phi; \phi') \quad (26)$$

which is analogous to the relation $Z_{ji} = -Z_{ij}^*$ which is valid for lossless $2n$ -ports in the impedance matrix theory.

B. Cyclic Symmetry

The cyclic symmetry of a 3-port requires the condition

$$G(\phi + 2\pi/3; \phi' + 2\pi/3) = G(\phi; \phi') \quad (27)$$

to be fulfilled.

C. Resonances of the Disk

The disk configuration with disconnected striplines has two types of resonances.

1) *Electric wall*: If the edge of the center conductor is short-circuited to the outer conductors, the resonance condition is that $E_z(R, \phi)$ is identically zero. Hence, the resonance condition is

$$G(\phi; \phi') = 0. \quad (28)$$

For the subject matter of this paper, this type of resonances is of no importance.

2) *Magnetic wall*: If the edge of the disk constitutes a magnetic wall, the resonance condition is

$$G(\phi; \phi') = \text{infinity}. \quad (29)$$

SIMPLIFIED BOUNDARY EQUATIONS

At this stage, it is advantageous to introduce two quantities ξ and Φ defined by

$$i\xi = 2\Psi G(-\pi/3; -\pi/3) \quad (30)$$

$$\Phi = 2\Psi G(-\pi/3; \pi/3). \quad (31)$$

The condition for losslessness (26) implies that ξ is real. In general, ϕ is complex. The application of the cyclic symmetry condition (27) to the coefficients of the right hand members of (20)–(22) and the substitution of (30) and (31) into these equations result in

$$A = i\xi a + \Phi b - \Phi^* c \quad (32)$$

$$B = -\Phi^* a + i\xi b + \Phi c \quad (33)$$

$$C = \Phi a - \Phi^* b + i\xi c. \quad (34)$$

MATCHING CONDITION

A good method of investigating the more special properties of the circulator is to terminate the output and the isolated port with reflectionless loads. Then, the parts of a wave incident upon the input port, which are dissipated in these terminations, are measures of the transmission coefficients. The reflected part of that wave is related to the reflection coefficient. In an earlier paper,⁵ this has been called the matching condition.

In this situation, only outward traveling waves are present in the output and isolated striplines so that, by virtue of (9), the relations

$$B/b = C/c = -\zeta \quad (35)$$

hold. Substitution into (32)–(34) and introduction of the complex quantity Θ given by

$$\Theta = \zeta + i\xi \quad (36)$$

result in the relations

$$A = \left(i\xi + \frac{\Phi^3 - \Phi^{*3} + 2\Phi\Phi^*\Theta}{\Phi\Phi^* + \Theta^2} \right) a \quad (37)$$

$$b = \frac{\Phi^2 + \Phi^*\Theta}{\Phi\Phi^* + \Theta^2} a \quad (38)$$

$$c = \frac{\Phi^{*2} - \Phi\Theta}{\Phi\Phi^* + \Theta^2} a. \quad (39)$$

SCATTERING MATRIX

The scattering matrix S of a cyclic symmetric 3-port is given by (1). If the electric and magnetic field intensities of the wave incident upon the input port are A_i and a_i , respectively, the following relations are valid:

$$A_i = \zeta a_i \quad (40)$$

$$A = (1 + \alpha) A_i \quad (41)$$

$$a = (1 - \alpha) a_i \quad (42)$$

$$\beta = B/A_i \quad (43)$$

$$\gamma = C/A_i. \quad (44)$$

After some algebraic manipulation, the following expressions are found for α , β and γ :

$$\alpha = \frac{\Phi^3 - \Phi^{*3} + \Phi\Phi^*(2\Theta - \Theta^*) - \Theta^2\Theta^*}{\Phi^3 - \Phi^{*3} + 3\Phi\Phi^*\Theta + \Theta^3} \quad (45)$$

$$\beta = -\frac{(\Phi^2 + \Phi^*\Theta)(\Theta + \Theta^*)}{\Phi^3 - \Phi^{*3} + 3\Phi\Phi^*\Theta + \Theta^3} \quad (46)$$

$$\gamma = -\frac{(\Phi^{*2} - \Phi\Theta)(\Theta + \Theta^*)}{\Phi^3 - \Phi^{*3} + 3\Phi\Phi^*\Theta + \Theta^3}. \quad (47)$$

All the elements of the scattering matrix are now expressed in terms of the Green's function $G(\phi; \phi')$ and the properties of the medium²¹ of the striplines ζ .

CIRCULATION CONDITION

From the considerations made above about cyclic symmetric 3-ports, such a 3-port is an ideal clockwise rotating circulator, if the conditions

$$\alpha = \gamma = 0 \quad (48)$$

$$|\beta| = 1 \quad (49)$$

are fulfilled. It will be clear that this only can be so if all media are lossless and all conductors are perfect. These conditions can be satisfied for given values of R and $4\pi M$ only with certain values of ω and H_i . Such a combination of the parameters R , ω , H_i and $4\pi M$ could be called a circulation adjustment.

The condition $\gamma=0$ is conceptually the same as the requirement $c=0$ for all a [cf., (39)]. In the latter form, it was earlier called⁵ the circulation condition. Since

$$\Theta + \Theta^* = 2\zeta \neq 0. \quad (50)$$

²¹ The formulas given apply only if the stripline medium is lossless.

application of $\gamma=0$ to (47) yields the circulation equation

$$\Theta = \Phi^{*2}/\Phi. \quad (51)$$

Substitution into (45) and (46) shows that $\alpha=0$ and

$$\beta = -\Phi/\Phi^* \quad (52)$$

which satisfy the other circulation requirements posed by (48) and (49). At first sight, one might wonder why $\gamma=0$ is a sufficient condition for circulator action, but it has been shown above that this is implied by losslessness and cyclic symmetry.

Rewriting the circulation equation (51) in its real and imaginary parts yields the two circulation equations (54) and (55) derived by Bosma.⁵ Therefore, the elements of the latter can be explained now in terms of the real and imaginary parts of the transfer wave impedances of the disk configuration and of the intrinsic wave impedance of the stripline medium.

GREEN'S FUNCTION

To solve the circulation equation in a particular case, Φ and Θ must be expressed in the characterizing quan-

$$E_{z,n}(r, \phi) = iA_n \frac{\zeta_{\text{eff}} J_n(kr) e^{in\phi}}{\frac{\kappa}{\mu} \frac{nJ_n(x)}{x} - J_n'(x)}, \quad (57)$$

where

$$x = kR = \omega \sqrt{\mu_0 \epsilon_0 \mu_{\text{eff}} \epsilon} R. \quad (58)$$

Let $H_\phi(R, \phi)$ be everywhere zero at the edge except at an azimuth ϕ' over a small angle $\Delta\phi'$, where it is $H_\phi(R, \phi')$. Then,

$$A_n = (1/2\pi) H_\phi(R, \phi') e^{-in\phi'} \Delta\phi' \quad (59)$$

and the corresponding contribution to $E_z(r, \phi)$ is

$$\Delta E_z(r, \phi) = \frac{i\zeta_{\text{eff}} H_\phi(R, \phi') \Delta\phi'}{2\pi} \sum_{n=-\infty}^{\infty} \frac{J_n(kr) e^{in(\phi-\phi')}}{\frac{\kappa}{\mu} \frac{nJ_n(x)}{x} - J_n'(x)}. \quad (60)$$

Comparing this with the defining relation (18) after some algebra, one obtains

$$G(r, \phi; R, \phi') = -\frac{i\zeta_{\text{eff}} J_0(kr)}{2\pi J_0'(x)} + \frac{\zeta_{\text{eff}}}{\pi} \sum_{n=1}^{\infty} \frac{\frac{\kappa}{\mu} \frac{nJ_n(x)}{x} \sin n(\phi - \phi') - iJ_n'(x) \cos n(\phi - \phi')}{\{J_n'(x)\}^2 - \left\{ \frac{\kappa}{\mu} \frac{nJ_n(x)}{x} \right\}^2} J_n(kr). \quad (61)$$

ties R, Ψ, ω, H_i and $4\pi M$. To that end $G(\phi; \phi')$ will now be deduced.

In the present case, the general solution of the Helmholtz equation (15) is a series in which the general term is given by

$$E_{z,n}(r, \phi) = a_n J_n(kr) e^{in\phi}, \quad (53)$$

where J_n is the n th order Bessel function. The corresponding term of the series for $H_\phi(r, \phi)$ is, by virtue of (16),

$$H_{\phi,n}(r, \phi) = i \frac{a_n}{\zeta_{\text{eff}}} \left[J_n'(kr) - \frac{\kappa}{\mu} \frac{nJ_n(kr)}{kr} \right] e^{in\phi}, \quad (54)$$

where ζ_{eff} is the effective intrinsic wave impedance of the ferrite as given by

$$\zeta_{\text{eff}} = \sqrt{\mu_0 \mu_{\text{eff}} / \epsilon_0 \epsilon}. \quad (55)$$

Inversely, if

$$H_{\phi,n}(R, \phi) = A_n e^{in\phi} \quad (56)$$

is the n th Fourier component of $H_\phi(R, \phi)$ at the edge, $r=R$ the corresponding term of the Bessel-Fourier series of $E_z(r, \phi)$ is given by

RESONANCES

Only the resonances of the disk configuration with magnetically short-circuited edge have importance for the present investigation. From (29), it is seen that such a resonance occurs if the denominator of one of the terms of the series (61) is zero,

$$J_n'(x) - \frac{\kappa}{\mu} \frac{nJ_n(x)}{x} = 0 \quad (62)$$

where n may be any positive or negative integer. From (54), it is easy to show that for these resonances $H_{\phi,n}(R, \phi)$ is identically zero. Moreover, for $n > 0$ the field pattern is rotating to the right and for $n < 0$ to the left. If the disk medium is isotropic, $\kappa/\mu=0$, the resonance conditions of the clockwise and anticlockwise rotating modes are identical

$$J_n'(x) = 0 \quad (63)$$

and, hence, these resonances are degenerate. When the gyrotropy increases, $\kappa/\mu \neq 0$, such a degenerate pair splits giving different x -values. For a disk R at a particular magnetic adjustment κ/μ and μ_{eff} , these two resonances have different frequencies which can be calculated from (58). With (62) and the recurrence relations of Bessel functions, it is easy to deduce that for a

small amount of gyrotropy $\kappa/\mu \ll 1$ the difference of x -values of a pair of resonances is determined by

$$(\Delta x)_{n,j} = 2 \frac{nx_{n,j}}{x_{n,j}^2 - n^2} (\kappa/\mu), \quad (64)$$

where $x_{n,j}$ is the j th solution of the n th order equation (63). n denotes the order of the resonant modes and j the number of the pair. In Fig. 4, the κ/μ -dependence has been sketched for several of these modes.

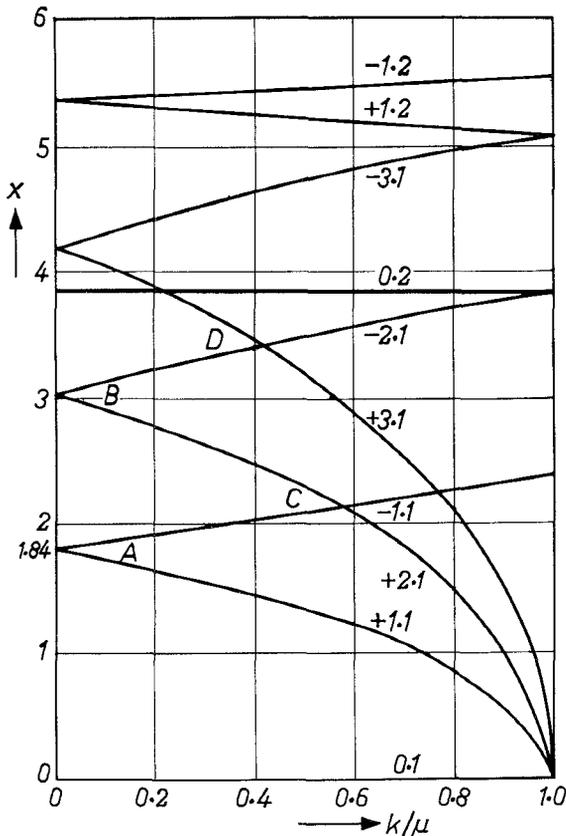


Fig. 4—The dependence on κ/μ of the x -values of several resonant disk modes. + sign denotes a clockwise rotating mode. - sign denotes an anticlockwise rotating mode. The first index refers to the order and the second one to the pair-number of a mode.

From experiments, it appeared that the practical circulation adjustment is situated between the first pair 1st-order resonances. In Fig. 4, this is in the neighborhood of *A*. Hence, the value of x is about equal to $x_{1,1}$. Therefore, it is supposed that the relation

$$J_1'(x_{1,1}) = 0, \quad x_{1,1} = 1.84 \quad (65)$$

is exactly valid for the circulator.

From the measured frequency difference between the first pair 1st-order resonances at the (magnetic) circulation adjustment and the value of $x_{1,1}$ calculated from it with (58), it appeared, by virtue of (64), that $\kappa/\mu = 0.125$. The other parameters, determined experimentally, of this circulation adjustment are $f = 450$ MHz, $R = 3.14$ cm, $H_i \approx 850$ Oe, $4\pi M = 1750$ Oe and $\Delta H \approx 150$ Oe.

The experimental value of κ/μ is very small, much smaller than would have been presumed intuitively. It is the opinion of the author that this is the main reason why an explanation of the junction circulation mechanism has been awaited for so long.

APPROXIMATE CIRCULATION ADJUSTMENT

Remembering the experimental data above, it is easy to see that the term for $n=1$ of the Green's function (61) is large compared with the other terms. Therefore, $G(r, \phi; R, \phi')$ will be approximated by retaining the term for $n=1$ only. A numerical estimation made afterwards showed that the error introduced in this way amounts to a few per cent only.

Substituting (65) into (61), by virtue of (19), for $G(\phi; \phi')$, the formula

$$G(\phi; \phi') = - \frac{x_{1,1} \zeta^{\text{eff}}}{\pi(\kappa/\mu)} \sin(\phi - \phi') \quad (66)$$

is found. Then, by virtue of (30), (31) and (36), Θ and Φ are found to be

$$\Theta = \zeta \quad (67)$$

$$\Phi = \frac{\sqrt{3} x_{1,1} \zeta^{\text{eff}}}{\pi(\kappa/\mu)} \Psi. \quad (68)$$

In this particular approximated case, both these quantities are real. Due to (9) and (55), substitution into the circulation equation (51) yields

$$\kappa/\mu = \frac{\sqrt{3} x_{1,1}}{\pi} \Psi \sqrt{\mu_{\text{eff}}/\epsilon}. \quad (69)$$

As $\omega \sqrt{\mu_0 \epsilon_0} = 2\pi/\lambda$, λ being the free space wavelength, (58) yields

$$R/\lambda = \frac{x_{1,1}}{2\pi \sqrt{\mu_{\text{eff}} \epsilon}}. \quad (70)$$

For small values of Ψ , this factor²² in the right hand member of (69) can be replaced by $(v/2R)$. Then, elimination of R from (69) and (70) yields the relation

$$\frac{\kappa/\mu}{\mu_{\text{eff}}} = \sqrt{3} \frac{v}{\lambda}. \quad (71)$$

FERRITE PROPERTIES

In order that the parameters R and H_i can be calculated at a given frequency, the electromagnetic properties of the ferrite, ϵ , κ , μ and μ_{eff} have to be known. Since the ferrite is biased far above the magnetic resonance

²² If, for the electrical field intensities at the ports, the averaged values and not the values of $E_z(R, \phi)$ at the centers of the striplines had been taken, the factor in Ψ in (69) would have been $\sin(\Psi)$. Then, the replacement by $(v/2R)$ is exact for all values of Ψ .

field $\kappa/\mu \ll 1$, the real parts of κ and μ need be considered only. These are given by

$$\mu = 1 + \frac{hm(h^2 - 1)}{(h^2 - 1)^2 - s^2} \quad (72)$$

$$\kappa = \frac{m(h^2 - 1)}{(h^2 - 1)^2 + s^2}, \quad (73)$$

where

$$m = 4\pi M/H_0 \quad (74)$$

$$h = H_i/H_0 \quad (75)$$

$$s = \Delta H/H_0. \quad (76)$$

For the circulators considered, the approximations $h^2 \gg 1$ and $h^2 \gg s$ can be made, so that (73) and (74) become

$$\mu = \mu_{\text{eff}} = (h + m)/h \quad (77)$$

$$\kappa = m/h^2. \quad (78)$$

CIRCULATION PARAMETERS

Insertion of (77) and (78) into (70) and (71) yields

$$R/\lambda = \frac{x_{1,1}}{2\pi\sqrt{\epsilon}} \sqrt{\frac{h}{h+m}} \quad (79)$$

$$h = \sqrt{\frac{\lambda m}{\sqrt{3}v}} - m, \quad (80)$$

while elimination of h from these equations results in

$$R/\lambda = \frac{x_{1,1}}{2\pi\sqrt{\epsilon}} \sqrt{1 - \sqrt{\sqrt{3}} \frac{mv}{\lambda}}. \quad (81)$$

From the last relation, one might conclude that small circulators can be obtained with large magnetizations and stripline widths. However, calculation of h shows that this quantity then becomes small (or even negative), which from considerations of resonance losses and magnetic saturation is not practical.

In general, it may be stated that in the UHF region h must be as large as possible. From (81), it is easily seen that h attains its maximum value for

$$m_{h_{\text{max}}} = h_{\text{max}} = (1/4\sqrt{3}) \frac{\lambda}{v}. \quad (82)$$

By virtue of (81), the corresponding relative disk radius is given by

$$(R/\lambda)_{h_{\text{max}}} = \frac{x_{1,1}}{2\pi\sqrt{2\epsilon}} = 0.207e^{-1/2}. \quad (83)$$

The inverse proportionality with $\sqrt{\epsilon}$ is self-evident. Increase of h_{max} by diminution of v (and corresponding choice of m) is not unrestricted, as the resulting stray fields become undesirably large.

Eqs. (80) and (81) can be rewritten in the form

$$H_i = \sqrt{\frac{\lambda}{\sqrt{3}v}} H_0 \cdot 4\pi M - 4\pi M \quad (84)$$

$$R/\lambda = \frac{x_{1,1}}{2\pi\sqrt{\epsilon}} \sqrt{\frac{H_i}{H_i + 4\pi M}}. \quad (85)$$

As H_0 is inversely proportional with λ , for the same stripline width and magnetization (v and $4\pi M$ constant), H_i and R/λ do not depend on frequency. Hence, with the latter increasing, H_i and R become too small with respect to H_0 and v , respectively. Consequently, v must be chosen smaller for higher frequencies. A good estimation is obtained by the assumption that H_i must be four times as large as H_0 at least, *i.e.*, $h > 4$. This number is rather arbitrary and it may be chosen smaller for the higher frequencies and small line widths. Due to (83), v is restricted in accordance with

$$v < \lambda/30, \quad (86)$$

so that for $\epsilon \approx 16$ a restriction of the kind

$$v/R < 0.75 \quad (87)$$

is valid, which is in agreement with (11).

Most of the experiments are performed with a manganese ferrite aluminate, the significant properties of which are $4\pi M = 1750$ Oe, $\Delta H = 150$ Oe and $\epsilon = 14.2$. The geometrical properties of the circulator were $v = 15$ mm and $u = 11$ mm. With these parameters, it can be calculated from (84) and (85) that $H_i = 935$ Oe and $R/\lambda = 0.046$. At a frequency of 450 MHz, one then finds theoretically $h = 5.82$, $\kappa/\mu = 0.112$, $R = 3.07$ cm and $v/R = 0.48$.

All approximations made above about h , κ/μ and v/R are justified. The agreement of these theoretical results with the experimental values quoted above is good. This good correspondence between theory and experiment is also seen in Fig. 5, where experimental and

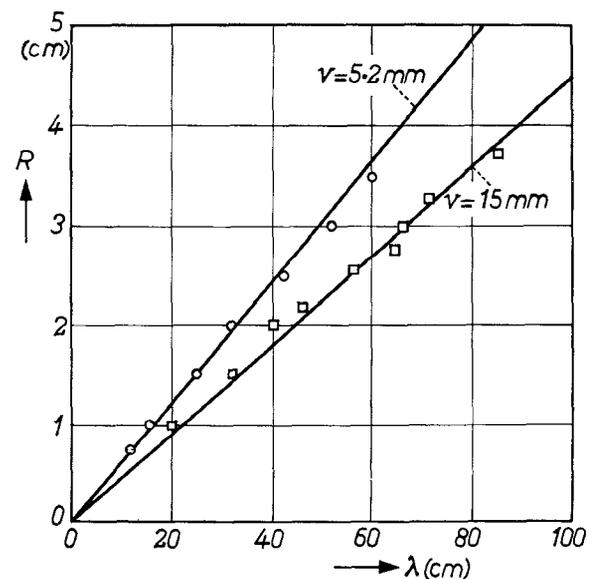


Fig. 5—Theoretical and experimental disk radii against free space wavelength with the stripline width as parameter. The drawn lines denote the theoretical results, (O) measured radii for $v = 5.2$ mm and (□) measured radii for $v = 15.0$ mm.

theoretical values of R are given as a function of λ , with values of v 15 mm and 5.2 mm, respectively. Values of H_i are not inserted into Fig. 5 as they were not measured accurately because of concentration of the static magnetic flux in the disks in most cases.

FREQUENCY DEPENDENCE

For any ferrite disk R , there is a circulation adjustment for one frequency only. This frequency will be called the circulation frequency ω_0 . If all other parameters are kept constant, the circulator characteristics are functions of frequency. Analogously, H_i can be varied in order to have a tunable circulator. The latter case will not be investigated here.

If $\omega \neq \omega_0$, $G(\phi; \phi')$ is a function of ω . Neglecting all terms for $n \neq 1$ again, from (61) it is

$$G(\phi; \phi') = \frac{Z_{\text{eff}} \left[\frac{\kappa}{\mu} \frac{J_1(x)}{x} \sin(\phi - \phi') - iJ_1'(x) \cos(\phi - \phi') \right] J_1(x)}{\pi \left[\{J_1'(x)\}^2 - \left\{ \frac{\kappa}{\mu} \frac{J_1(x)}{x} \right\}^2 \right]} \quad (88)$$

For $x = x_{1,1}$ and $\omega = \omega_0$ it is easy to deduce that μ does not depend on ω and that the relations

$$\frac{d(\kappa/\mu)}{d\omega} = \frac{\kappa/\mu}{\omega_0}, \quad \frac{dx}{d\omega} = \frac{x_{1,1}}{\omega_0} \quad (89)$$

hold. Using them together with recurrence relations of Bessel functions and (65) and (66), $G(\phi; \phi')$ can be developed into a Taylor-series. Neglecting 2nd and higher order terms of $\delta\omega$, where

$$\delta\omega = \omega - \omega_0, \quad (90)$$

this series is given by

$$G(\phi; \phi') = -\frac{\zeta_{\text{eff}}}{\pi} \left[\frac{x_{1,1}}{\kappa/\mu} \sin(\phi - \phi') + i \frac{x_{1,1}(x_{1,1}^2 - 1)}{(\kappa/\mu)^2 \omega_0} \cos(\phi - \phi') \delta\omega \right]. \quad (91)$$

Then, by virtue of (30), (31) and (36), the quantities Θ and Φ are found to be

$$\Theta = \zeta(1 - i2\eta) \quad (92)$$

$$\Phi = \zeta(1 + i\eta), \quad (93)$$

where

$$\eta = \frac{x_{1,1}^2 - 1}{\sqrt{3}(\kappa/\mu)} \frac{\delta\omega}{\omega_0}. \quad (94)$$

Substitution into (45)–(47) yields, for the elements of the scattering matrix,

$$\alpha = \frac{-3\eta^2 + i2\eta}{(4 - 9\eta^2) - i6\eta} \quad (95)$$

$$\beta = \frac{(-4 + 6\eta^2) + i2\eta}{(4 - 9\eta^2) - i6\eta} \quad (96)$$

$$\gamma = \frac{b\eta^2 + i2\eta}{(4 - 9\eta^2) - i6\eta}. \quad (97)$$

Eqs. (3) and (4) are easily verified. For small η or, what is the same, for small frequency shifts, $\delta\omega/\omega_0 \ll 1$, α , β and γ can be approximated by

$$\alpha = \gamma = i\eta/2 = i \frac{x_{1,1}^2 - 1}{2\sqrt{3}(\kappa/\mu)} \frac{\delta\omega}{\omega_0} \quad (98)$$

$$1 - |\beta|^2 = \frac{1}{2} \eta^2 = \frac{1}{2} \left\{ \frac{x_{1,1}^2 - 1}{\sqrt{3}(\kappa/\mu)} \right\}^2 \left(\frac{\delta\omega}{\omega_0} \right)^2. \quad (99)$$

α and γ measure the return losses and the isolation, while $(1 - |\beta|^2)$ is an expression for the forward attenua-

tion. Due to dissipative losses, the latter is much larger in practice. The relation $\alpha = \gamma$ is easy to verify experimentally.

BANDWIDTH

The bandwidth of a circulator is not an unambiguous quantity. It can be defined in several ways. A practically useful method is the specification of a maximal value for the modulus of the reflection coefficient. Thus,

$$|\alpha| \leq \rho_{\text{max}}. \quad (100)$$

The isolation is limited at the same time. Substitution of (98) yields, for the relative bandwidth,

$$\frac{\Delta\omega}{\omega_0} = \frac{4\sqrt{3}}{x_{1,1}^2 - 1} (\kappa/\mu) \rho_{\text{max}}. \quad (101)$$

For $\rho_{\text{max}} = 0.1$ (which is the same as a VSWR of 1.22, an isolation of 20 db and a minimal insertion loss of 0.1 db) at 450 MHz, a bandwidth of 3.2 per cent is found. Experimentally, it appeared to be 4.1 per cent. Why the experimental bandwidth is larger than the theoretical one can be understood from the fact that the dissipative losses, which tend to increase the bandwidth, are not considered in the theory.

From (101), it can be seen that the bandwidth is proportional to the amount of gyrotropy κ/μ . This can be understood from an inspection of (62) or (64). The disturbance of the original isotropic resonance is proportional to κ/μ . The larger κ/μ is, the more distant are the resonant clock- and anticlock-wise rotating modes. Hence, in the frequency region between the latter, the dependence of several quantities on frequency will be smaller, resulting in a larger bandwidth. Although, in most cases, a large bandwidth is desirable, κ/μ cannot

be chosen large, as a small H_i would be the result which is not permitted because of the losses due to it. However, it is shown below that another straightforward method exists to obtain large bandwidths.

INPUT IMPEDANCE

Let the characteristic impedance of the striplines be Z and the output and isolated ports be reflectionlessly terminated. Then, upon neglecting higher order terms of $\delta\omega/\omega_x$ again, the input impedance Z_i is easily derived. The result is

$$Z_i = R_i + iX_i \quad (102)$$

with

$$R_i = Z \quad (103)$$

$$X_i = Z \frac{x_{1,1}^2 - 1}{\sqrt{3}(\kappa/\mu)} \frac{\delta\omega}{\omega_0} \quad (104)$$

The real part of Z_i , R_i does not depend on frequency, but the imaginary part X_i does. Remembering that the time dependent factor of the field is $\exp(-i\omega t)$, one can see that, for $\omega < \omega_0$, X_i is inductive and, for $\omega > \omega_0$, capacitive. This parallel resonant circuit behavior is confirmed by experiment.

BANDWIDTH ENLARGEMENT

From the dependence of the input impedance on frequency, a method of enlarging the bandwidth enormously can be proposed. For a lossless, cyclic-symmetric 3-port circulator, the isolation is related directly to the reflection coefficient [cf., (7)]. When the latter is zero, the former is infinite. Now, α can be made zero over a rather large bandwidth and, hence, so can γ . This can be accomplished with a lossless series circuit of an inductance L and a capacitance C . The series impedance of such a circuit is

$$Z_{\text{series}} = i \left(\frac{1}{\omega C} - \omega L \right) \quad (105)$$

Choosing L and C so that the relations

$$\omega_0 = 1/\sqrt{LC}, \quad (106)$$

$$2\omega L_0 = Z \frac{x_{1,1}^2 - 1}{\sqrt{3}(\kappa/\mu)} \quad (107)$$

hold and connecting such resonant LC -circuits in series with the three ports, the reflections are eliminated over a rather large bandwidth centered at the circulation frequency.

In this way, with small coils and capacitors, it was found possible to make a circulator having a bandwidth of about 30 per cent. From 310 MHz up till 420 MHz, it had a maximal VSWR of 1.2, a minimal isolation of 19.5 db and a maximal insertion loss of 1.1 db. This last

figure is a little high, but it is believed that it can be improved by using a better ferrite.

FIELD DISTRIBUTION

It is easy to show that at the circulation adjustment the relations

$$b = a, \quad c = 0 \quad (108)$$

hold [cf., (10)] and that, upon substitution into (18), they yield, for the electric field intensity in the disk, the 1st order approximation

$$E_z(r, \phi) = \frac{2x_{1,1}Z_{\text{eff}} \sin \Psi J_1(kr) \sin \phi}{\pi(\kappa/\mu)J_1(x_{1,1})} a. \quad (109)$$

Insertion of this result into (16) and (17) gives for the magnetic field intensity in the disk the approximate expressions

$$H_\phi(r, \phi) = - \frac{2x_{1,1} \sin \Psi}{\pi(\kappa/\mu)J_1(x_{1,1})} \cdot \left[\frac{\kappa}{\mu} \frac{J_1(kr)}{kr} \cos \phi - iJ_1'(kr) \sin \phi \right] a \quad (110)$$

$$H_r(r, \phi) = - \frac{2x_{1,1} \sin \Psi}{\pi(\kappa/\mu)J_1(x_{1,1})} \cdot \left[-\frac{\kappa}{\mu} J_1'(kr) \sin \phi + i \frac{J_1(kr)}{kr} \cos \phi \right] a. \quad (111)$$

Inspection of these formulas shows that the magnetic field intensity is anticlockwise rotating and elliptically polarized. The axis ratio is κ/μ . Calculation of the high frequency part of the magnetization shows that it is clockwise rotating and also elliptically polarized with an axis ratio $\kappa/\mu/(\mu_{\text{eff}}-1)$.

The electric field has been measured, too. The results of this experiment are sketched in Fig. 6, in which the lines of equal amplitude and the constant phase lines have been drawn. Comparison of the experimental results with the expression given above shows that the agreement is good, except at those places where the latter is small; that is, along the diameter $\phi = 0, \pi$, which is symmetrically situated between the input and the output port. This is quite understandable as in those places the neglected terms for $n \neq 1$ give a non-negligible contribution to the total electric field intensity and spread the phase lines.

Retaining 18 terms of the series involved, a solution of the same problem, as it is stated by Bosma,⁵ has been determined with an electronic computer. Indeed, the field distribution so obtained agrees much better with the experimental result of Fig. 6. The circulation parameters of this "accurate" solution are $x = 1.852$, $h = 6.034$, $H_i = 970$ Oe, $4\pi M = 1750$ Oe, $\kappa/\mu = 0.109$, $f = 450$ MHz and $R = 2.90$ cm. These data, too, correspond well to those quoted above.

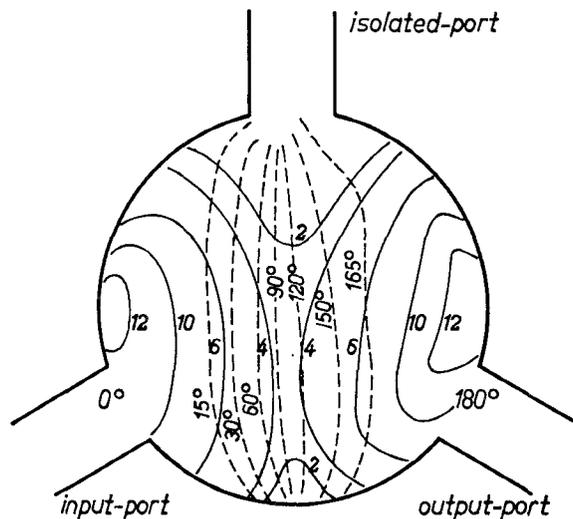


Fig. 6—Measured amplitude and phase of the electric field in the circulator disks. Drawn lines indicate the lines of equal amplitude. Broken lines are equal phase-lines. (The amplitude is given in arbitrary units.)

THE CIRCULATION MECHANISM

The primary condition for circulation seems to be that the adjustment is in the neighborhood of the degeneracy of two resonances of the disk configuration (with disconnected striplines). If these resonances form a pair of the same order, as is the case with the solution found here, the implication of (64) is that κ/μ is small. This is also confirmed by Butterweck¹⁶ who studied a model of the 3-port waveguide junction circulator, consisting of a circular-cylindrical cavity with an axially biased, concentric, thin and full height post of ferrite and three rectangular waveguides coupled weakly to the cavity at equal distances around the cylindrical wall. Butterweck also found that circulator action was possible near the degeneracy of the two first-order oppositely rotating modes. Although, in his case, κ/μ in the ferrite is not small, the effective gyrotropy, as averaged over the whole cavity volume, is.

The first order resonance of the isotropic disks has a distribution of the electric field as sketched in Fig. 7(a) at $t=0$. It is antisymmetric with respect to the symmetry diameter. A quarter of a period later $t=t/4$, the electric field is identically zero but, then, the magnetic field attains its maximum. The lines of the latter have been drawn in Fig. 7(b). If, instead of being isotropic, the disks are weakly gyrotropic, the magnetic field present at $t=T/4$ will induce a magnetic dipole distribution. This has been sketched in Fig. 8(b). As the disks are polarized perpendicularly, these dipoles rotate to the right. A quarter of a period earlier $t=0$, they were directed as it has been indicated in Fig. 8(a). The magnetic induction then being zero, they cause a demagnetizing field which, too, has been sketched in Fig. 7(a). The latter is in phase (or counterphase) with the electric field and it is symmetric with respect to the symmetry diameter. Hence, at the edge $r=R$, a real, radial power flow exists. In four regions, this power flow is alternately

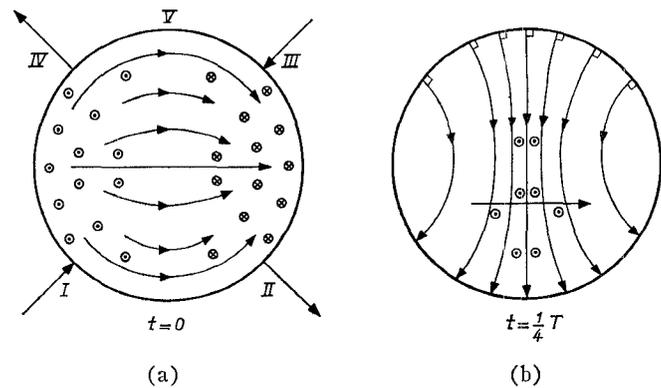


Fig. 7—The electromagnetic field in the circulator disks at times (a) $t=0$ and (b) $t=T/4$, respectively. The circles denote the electric field intensity, the drawn lines the magnetic field intensity and the straight arrows the power flow.

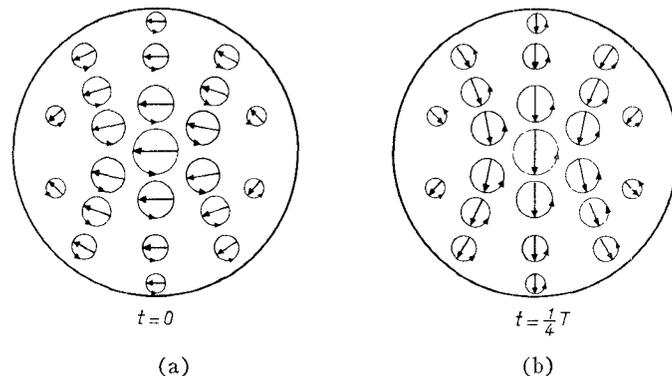


Fig. 8—The magnetic dipole distribution in the circulator disks at times (a) $t=0$ and (b) $t=T/4$, respectively.

inwardly and outwardly directed. In Fig. 7(a), this has been indicated with four radial arrows. Because of the resonant character of the isotropic (symmetric) part of the magnetic field, the reactive power flow at the edge is zero.

If, in two successive regions (I and II), waveguides (striplines) are connected, the power radiated inwardly and outwardly, respectively, can be transported via these guides. If, in the other two regions (III and IV), no guides are present, no radial current can flow from the center conductor, no power can be radiated and, consequently, the azimuthal component of the magnetic field must there be zero. This can be so if only other modes are excited. As these modes are not resonant, they do not contribute substantially to the electric field at the edge, but they do near the symmetry diameter. The (small) electric field excited in this way will be in phase with the magnetic field there [Fig. 7(b)], so that across the symmetry diameter a power flow occurs which transports the energy from region I towards region II.

Due to the excitation of the other modes, $n \neq 1$ everywhere at the edge except at the regions I and II, the azimuthal magnetic field is zero. Because near V [Fig. 7(a)] the electric field is also zero, connection of a waveguide at that place does not disturb the fields

and, hence, no power exchange occurs there. This guide is isolated.

Finally, it will be clear that the wave impedances of the guides at I and II have to be matched to the ratios of the electric and magnetic field intensities in the ferrite at those places. If R is chosen such that the isotropic disks are resonant, κ/μ and μ_{eff} have to be adjusted by means of H , so that the guides are matched to the disks. Mathematically, (79) is the expression of the former condition and (80) that of the latter.

It will be clear that the effective electrical length of the circulator is equal to half a wavelength.

APPLICATION IN THE UHF REGION

Three reasons exist why the 3-port stripline Y -circulator polarized above resonance is advantageously used at frequencies below 1000 Mc/sec.

First, it is known from resonance isolators at the lower kMc-frequencies that thin, flat forms of ferrite magnetized perpendicularly are favorable with respect to saturation and, hence, for small low-field losses. The same is valid for the circulator in question.

Second, nearly all known nonreciprocal devices are of the same order of largeness as the free space wavelength, at least in one dimension. This signifies that in the UHF region the construction would become big and voluminous. However, the diameter of the stripline Y -circulator is determined mainly by the wavelength in the ferrite. By virtue of (84) $\epsilon = 14-16$, the latter is an order of magnitude smaller than the free space wavelength as is, therefore, the circulator as well.

The last and most important reason is the fact that the circulator in question requires essentially a very high bias field $h > 4$. It is much higher than it is in the case of any one of the well-known principles of nonreciprocal action. And, as has been argued above, this high bias field is very favorable for small insertion losses, which are usually difficult to obtain in the UHF region.

OTHER SOLUTIONS OF THE PROBLEM

Any solution of the circulation equation (51) may result in conditions for circulator adjustment. Such solutions can be found in the neighborhood of degeneracies of disk resonances. This has been confirmed by Cohen and Davies¹¹ who computed electronically, from (54) of Bosma,⁵ curves in the κ/μ , x -plane along which circulator action is possible. The straight line $x = 1.84$ for $\kappa/\mu < 0.25$ coincides nearly with one of these curves. Other curves join at the degeneracies B , C and D (Fig. 4) and are between the mode pairs $(-2, 1-2, 1)$, $(-1, 1-2, 1)$ and $(-2, 1-3, 1)$, respectively. To give a complete picture, for small values of κ/μ , they have also found a curve within the region ABC , which seems not to be related to a degeneracy. Moreover, the separate curves are linked crossing over a resonant mode curve

somewhere between two degeneracies, *e.g.*, over the curve $-1, 1$ between A and C . However, it may appear that these solutions do not apply to practical circulators and, in many cases, they do not.

In fact, circulator action has been detected experimentally somewhere near B (Fig. 4). However, using the second order resonance is not a practical proposition, since due to the large x -value the radius is large. Furthermore, relative to the transmitted power, much of its field energy is stored in the disk, so that the insertion loss is relatively large. It is the opinion of the author that many rectangular waveguide junction circulators can be improved because, in many of them, higher order resonances are employed.

The solution evaluated by Bosma⁵ is near the degeneracy of the first anticlockwise rotating 1st order mode and the first clockwise rotating 2nd order mode which is in the neighborhood of C in Fig. 4. The mathematical difficulties of the evaluation of this solution are much greater than of the solution presented in this paper, because the gyrotropy is not small ($\kappa/\mu \approx 0.5$) and it is not permitted to neglect all terms of the series involved except the two resonant terms. Besides, it has turned out that the approximation $J_0(x) = 0$ made by Bosma⁵ is not quite justified. But, although the solution near C (Fig. 4) is mathematically nearly possible and would require a small disk radius, it is not a practical proposition since κ/μ is not very small; hence, as h is not large, $h \approx 1.5-2.0$ in the UHF region, resonance (and perhaps low field) losses are not small. In fact, the resonances of the disk configuration could not be detected even at values of κ/μ larger than about 0.3, because they were masked by losses. The agreement between the theoretical and the experimental radii as a function of frequency found by Bosma⁵ must be looked upon as merely an awkward coincidence. If at the time the field distribution in the disk had been measured, the practically false character of the solution would have been observed. Now, the conclusions drawn from it may be considered obsolete.

The theory developed in this paper applies also to circulators which are biased below resonance, if the section about the magnetic properties of the ferrite and its consequences are restated. Moreover, an analogous analysis can probably be carried out for the rectangular waveguide junction circulator.

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