

obtaining a spectral match of the laser emission to the maser pump transition is concerned. On the other hand, the liquid nitrogen cooled laser emission is broader than the liquid helium cooled laser when driven above threshold. Consequently, a somewhat more severe limitation arises as to the amount of maser pump power which can actually be coupled into the maser ruby if a nitrogen cooled laser is employed.

The magnetic fields required in this scheme are feasible with superconducting solenoids which have critical temperatures below the higher frequency maser operating temperature, however. Consequently, we envisage the maser cryogenic system to consist of a triple dewar, *i.e.*, an outer nitrogen jacket, a second helium jacket for the superconducting magnet, and an inner cold chamber containing the maser ruby and a heater to control the

temperature of the maser. A schematic depicting such apparatus is shown in Fig. 5.

#### CONCLUSION

A number of assumptions are required in outlining a procedure for operating the laser-pumped ruby maser at the highest possible frequency. Little directly related experimental data can be used to support these assumptions, and we may therefore only judge that the laser-pumped ruby maser appears feasible at a frequency in the neighborhood of 300 Gc/sec. Finally, we have confined our discussion to ruby, since we wished to be as concrete as possible in speculation. The study of other, either extant or forthcoming, three-level laser materials may well allow better methods of obtaining a laser-pumped maser than considered here.

## The Re-Entrant Cross Section and Wide-Band 3-db Hybrid Couplers\*

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**Summary**—A new type of parallel-coupled TEM-mode cross section is described and named the re-entrant cross section. An analysis of the even- and odd-mode characteristic impedances of the re-entrant cross section shows it to have advantages in the case of tight coupling when compared to previously used parallel-strip cross sections. Close tolerances are easily held, and considerable misalignment is permissible. Two single-section 3-db couplers were tested with coupling curves very close to theoretical, and with good directivity. Then a three-section coupler having a re-entrant center section was designed for the 400- to 2000-Mc band, yielding a coupling variation less than 0.4 db and a minimum directivity of 29 db. Next a three-section coupler was designed for the 1- to 5-Gc band. A series of modifications resulted in a final model having a coupling variation within 0.5 db, and a minimum directivity of about 22 db.

#### INTRODUCTION

**B**ACKWARD-COUPLING parallel strip line couplers can theoretically be made to have nearly constant coupling over very large bandwidths by properly connecting coupled quarter-wave sections in cascade.<sup>1-3</sup> For example, one section yields 2:1 band-

width with  $-3 \pm 0.3$ -db coupling, while three sections in a symmetrical configuration can be designed to yield 3:1 bandwidth with  $-3 \pm 0.1$ -db coupling, or 5.1:1 bandwidth with  $-3 \pm 0.4$ -db coupling.<sup>2</sup> However, when using more than one section, discontinuity effects have previously reduced the coupler's directivity and increased its VSWR. Furthermore, it has been difficult to maintain sufficiently tight dimensional tolerances in the very closely spaced strips of the center section. Because of these problems, prior work on three section 3-db couplers for 3:1 bands has resulted in low directivity.<sup>2</sup> The situation is even more severe for designs of greater bandwidth.

In order to alleviate the problems of discontinuity effects and tolerances, different configurations have been evolved that include a new TEM-mode coupling cross section (patent pending). This cross section, which was named the *re-entrant* cross section, offers a number of advantages: 1) its design equations are simple; 2) by its nature tight tolerances are easily held; 3) it can tolerate considerable misalignment of its parts; and 4) experience has shown that it may be joined with conventional strip line end sections without excessive discontinuity effects.

#### DESIGN FORMULAS FOR RE-ENTRANT LINE COUPLERS

Fig. 1 shows the new re-entrant line coupling cross section. Conductors *A* and *B* are coaxial line center con-

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<sup>1</sup> B. M. Oliver, "Directional electromagnetic couplers," *Proc. IRE*, vol. 42, pp. 1686-1962; November, 1954.

<sup>2</sup> J. K. Shimizu, "Strip-line 3-db directional couplers," 1957 IRE WESCON CONVENTION RECORD, Pt. 1, pp. 4-15.

<sup>3</sup> J. K. Shimizu and E. M. T. Jones, "Coupled-transmission-line directional couplers," *IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES*, vol. MTT-6, pp. 403-410; October, 1958.

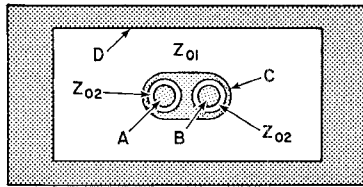


Fig. 1—Re-entrant coupled cross section.

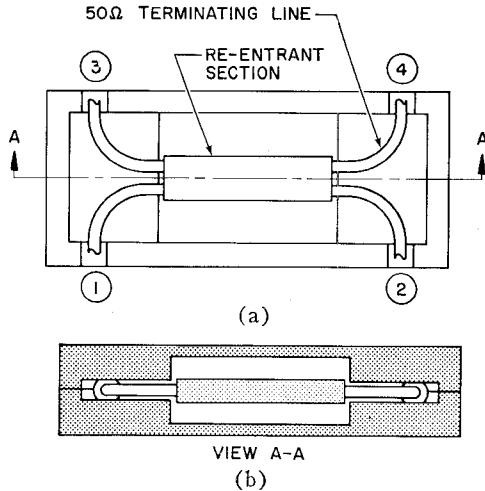


Fig. 2—Views of re-entrant quarter-wave directional coupler. (a) Top view, cover removed. (b) Longitudinal section.

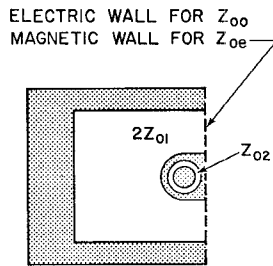


Fig. 3—Boundary conditions for even- and odd-modes in bisected re-entrant coupled cross section.

ductors of characteristic impedance  $Z_{02}$  within the intermediate conductor,  $C$ , while  $C$  is the center conductor of a transmission line of characteristic impedance  $Z_{01}$  within  $D$ . One method of connecting conductors  $A$  and  $B$  to the output ports is shown in Fig. 2. Note that the intermediate conductor is floating electrically. At first glance it appears that  $A$  and  $B$  in Fig. 1 are completely shielded from each other by  $C$ . However, the transmission line between  $C$  and  $D$  is in series with the two coaxial lines contained in  $C$ , and acts as the mutual coupling medium.

As in the case with conventional directional couplers, the re-entrant line coupler is best analyzed in terms of its even- and odd-mode characteristic impedances.<sup>4</sup> The even-mode characteristic impedance occurs between  $A$  and  $D$  when the structure is bisected through its vertical

plane of symmetry by a magnetic wall, while the odd-mode characteristic impedance occurs when the bisecting plane is an electric wall.

Fig. 3 shows these two cases. For the even mode,  $Z_{oe}$  is equal to  $Z_{02}$  and  $2Z_{01}$  in series

$$Z_{oe} = Z_{02} + 2Z_{01}. \tag{1}$$

For the odd mode,  $Z_{01}$  is short-circuited by the electric wall, so that

$$Z_{oo} = Z_{02}. \tag{2}$$

As with other forms of parallel transmission line directional couplers, this new structure is a backward-directed coupler. That is, if Port 1 in Fig. 2 is the input port, Port 3 will be the coupled port and Port 4 the isolated port. If discontinuity reactances are neglected, the isolation and input match will be perfect when the following condition is satisfied.

$$Z_o = \sqrt{Z_{oe}Z_{oo}} \tag{3}$$

where  $Z_o$  is the characteristic impedance of the terminating lines. The coupling factor is as follows at center frequency:

$$k = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} = \frac{Z_{o1}}{Z_{o1} + Z_{o2}}. \tag{4}$$

In the design of a directional coupler, the desired value of  $k$  will first be determined. Then  $Z_{oe}$  and  $Z_{oo}$  may be calculated as follows:

$$Z_{oe} = Z_o \sqrt{\frac{1+k}{1-k}} \tag{5}$$

$$Z_{oo} = Z_o \sqrt{\frac{1-k}{1+k}}. \tag{6}$$

The necessary values of  $Z_{o1}$  and  $Z_{o2}$  are

$$Z_{o1} = \frac{1}{2}(Z_{oe} - Z_{oo}) = Z_o \left( \frac{k}{\sqrt{1-k^2}} \right) \tag{7}$$

$$Z_{o2} = Z_{oo} = Z_o \sqrt{\frac{1-k}{1+k}}. \tag{8}$$

Despite their simplicity, all of the above equations are exact for any value of  $k$ .

### SINGLE-SECTION DESIGN

A quarter-wave section directional coupler will couple within  $-3 \pm 0.3$  db over an octave bandwidth if it is designed for a coupling value of  $-2.7$  db ( $-2.69$  db to be exact) at center frequency. In that case let  $k = 0.734$  and  $Z_o = 50$  ohms. By means of (7) and (8), the characteristic impedances in the re-entrant line cross section are found to be  $Z_{o2} = 19.6$  ohms and  $Z_{o1} = \frac{1}{2}(127.8 - 19.6) = 54.1$  ohms.

In two quarter-wave models that were constructed,

<sup>4</sup> E. M. T. Jones and J. T. Bolljahn, "Coupled strip-transmission line filters and directional couplers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 75-81; April, 1956.

the common outer conductor of the two coaxial lines consists of two pieces of 0.250-inch O.D. tubing soldered together with solder filling the spaces between the tubes. Thus, the outer perimeter is 0.250 inch thick and 0.500 inch wide. The wall thickness of the tubing is 0.022 inch, so that the inner diameter of the outer conductor is 0.206 inch. The ratio of conductor diameters to yield 19.6 ohms is 1.386, giving an inner conductor diameter of 0.149 inch. The gap between the conductors is thus about 0.028 inch. The inner conductor rods are supported by teflon rings one-sixteenth inch long.

Design data was not available for the  $Z_{o1}$  geometry. Therefore, the assumption was made that the  $Z_{o1}$  center conductor cross section is equivalent to a rectangular strip cross section having the same thickness and the same area. Thus if  $d_o$  is the outer diameter of the tubing,  $t = d_o$  and  $w = 1.786d_o$ . The accuracy of this equivalence is believed to be sufficiently good for this purpose. Then with the aid of thick strip characteristic impedance data,<sup>5,6</sup> it was determined that  $Z_{o1} = 54.1$  ohms for  $d_o/b = 0.335$ . Thus for  $d_o = 0.25$  inch,  $b = 0.746$  inch.

Fig. 4 is a photograph of the internal arrangement of parts of the first experimental single section 3-db coupler, which was designed for  $f_o = 500$  Mc. Fifty-ohm slab line joins the re-entrant section to the connectors. The measured response in Fig. 5 shows that very good coupling accuracy and high directivity were achieved. The measurements were extended upwards in frequency in order to illustrate the periodic response of this type of coupler. The coupling and directivity in the band centered at  $3f_o$  are essentially the same as that centered at  $f_o$ .

After the results of Fig. 5 were obtained, a second re-entrant coupler was designed for the 1000- to 2000-Mc band with  $f_o = 1500$  Mc. This coupler was made identical to the first coupler, except that the coupling region was made about one-third as long. Its mid-band coupling turned out to be  $-2.6$  db, which is 0.1 db greater than the design value. The minimum directivity in the 1000- to 2000-Mc band is 29 db. The response curves are plotted in Fig. 6.

### THREE-SECTION DESIGN

The section couplings necessary to achieve a 5.1:1 bandwidth with  $-3 \pm 0.4$  db over-all coupling in a symmetrical three-section design are  $k = 0.8612$  ( $-1.28$  db) and  $k' = 0.2337$  ( $-12.64$  db), where  $k$  is the coupling factor of the center section and  $k'$  that of the end sections. With  $Z_o = 50$  ohms, the even- and odd-mode characteristic impedances of these sections are, respectively,  $Z_{oe} = 183$  ohms,  $Z_{oo}' = 63.5$  ohms,  $Z_{oo} = 13.66$  ohms, and  $Z_{oo}' = 39.4$  ohms. The re-entrant structure has been

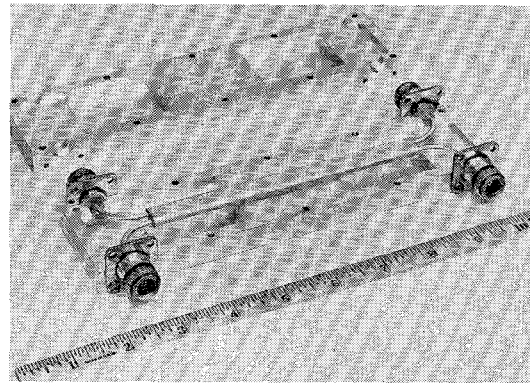


Fig. 4—Photograph of single section re-entrant line coupler for 340–680 Mc band.

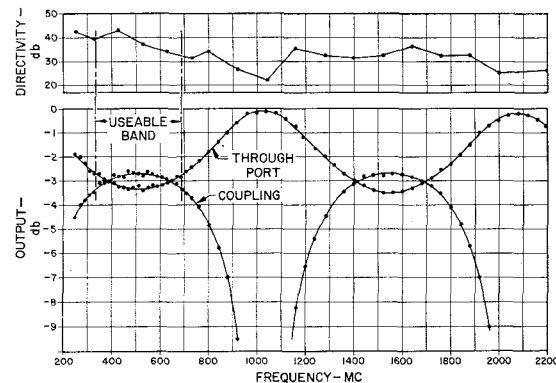


Fig. 5—Coupling and directivity response of quarter-wave re-entrant coupler after modification ( $f_o = 500$  Mc).

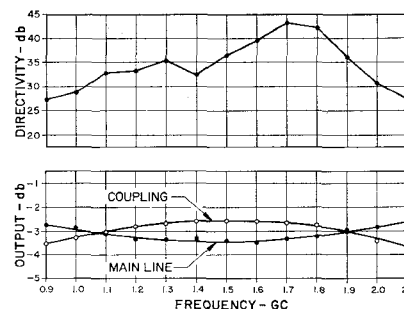


Fig. 6—Response of a 1- to 2-Gc single section re-entrant directional coupler.

found suitable for the center section, but conventional parallel-coupled strip line is better for the end sections. The center-section characteristic impedances are obtained from (7) and (8) as follows:  $Z_{o1} = 84.7$  ohms and  $Z_{o2} = 13.66$  ohms. The dimensions were computed in the same way as in the case of the quarter-wave coupler. Based on a tubing outer diameter of 0.1875 inch and inner diameter 0.143 inch, the coaxial center conductor diameters are 0.114 inch, and the plate spacing is 0.956 inch.

The end sections were designed for copper-clad Tellite 3A dielectric material, which has a dielectric constant of 2.32. The thickness of each board is 0.125 inch so that the ground plane spacing is 0.250 inch. The copper thickness is 0.0014 inch. By means of published for-

<sup>5</sup> S. B. Cohn, "Problems in strip transmission lines," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 119-126; March, 1955.

<sup>6</sup> R. H. T. Bates, "The characteristic impedance of the shielded slab line," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 28-33; January, 1956.

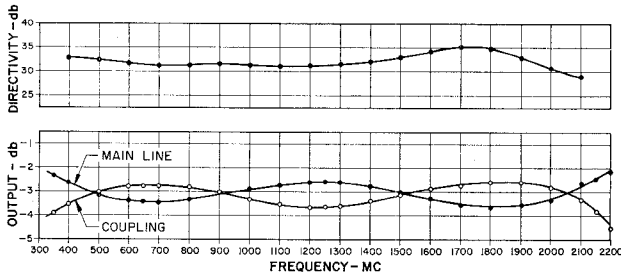


Fig. 7—Response curves of 400-2000 Mc three section directional coupler with re-entrant center section, after modifying end sections.

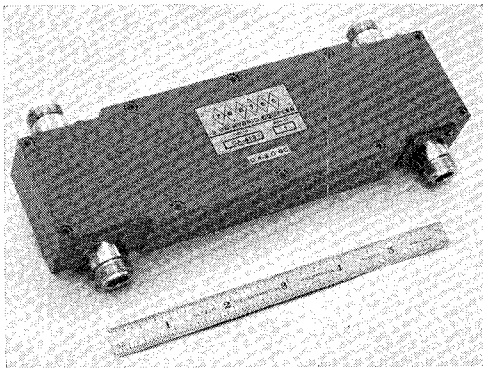


Fig. 8—Photograph of completed 400-2000-Mc hybrid coupler.

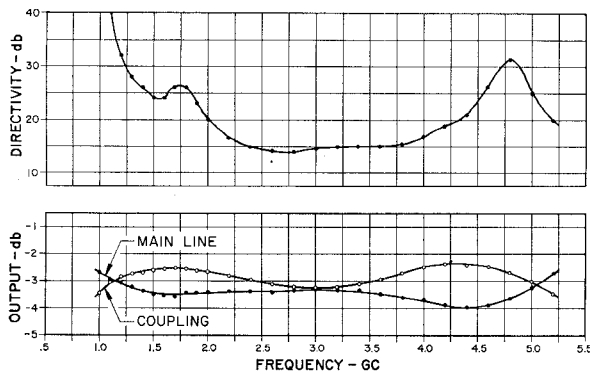


Fig. 9—First response for 1-5 Gc coupler.

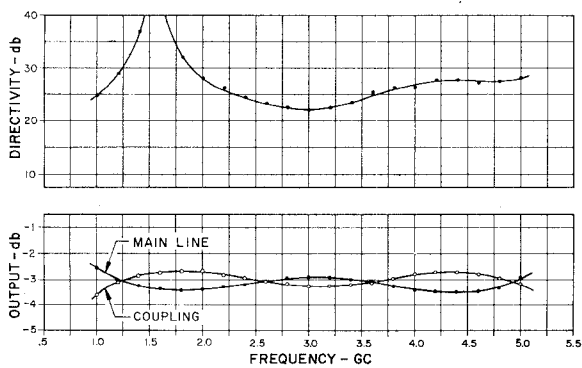


Fig. 10—Final data on 1-5 Gc hybrid coupler.

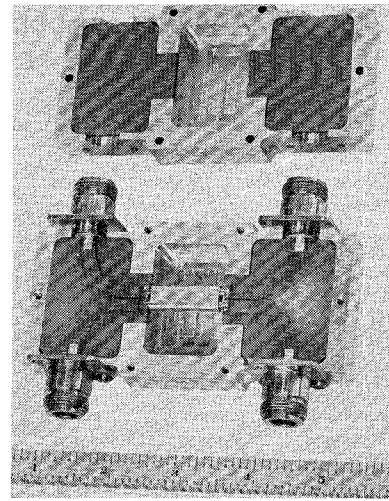


Fig. 11—Internal view of final 1-5 Gc hybrid coupler.

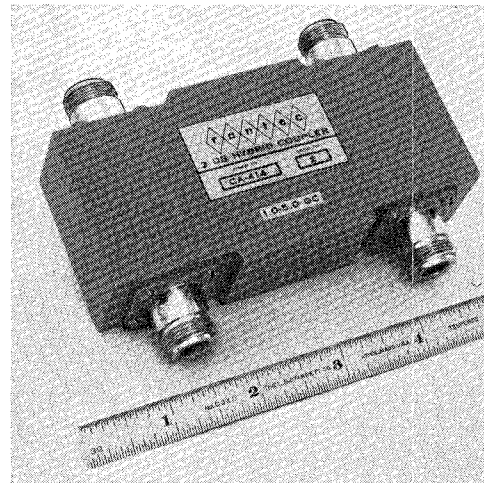


Fig. 12—View of completed 1-5 Gc hybrid coupler.

mulas,<sup>7</sup> the essential dimensions were computed in terms of the desired  $Z_{oe}$  and  $Z_{oo}$  values, taking thickness into account. In the coupled region, the strip width is  $w=0.173$  inch and the gap is  $s=0.0253$  inch. The 50-ohm terminating strips are 0.194 inch wide.

A coupler designed for the 400-2000 Mc band was built and tested. The first test revealed a fair conformance to the desired coupling response, but poor directivity. After some minor configuration changes near the ends of the end section strips to cancel discontinuity effects, the good results of Fig. 7 were achieved. A photograph of the completed model is shown in Fig. 8.

It was next decided to construct a model for 1 to 5 Gc having the same cross section dimensions as the 0.4- to 2.0-Gc model, but with lengths reduced in a 2.5-to-1 ratio. The measured curves for the first 1-5 Gc three-section coupler are shown in Fig. 9. It is evident that the coupling is too great, and that it increases gradually

<sup>7</sup> S. B. Cohn, "Shielded couple-strip transmission lines," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 29-38; October, 1955.

with frequency. This effect may be attributed to discontinuity effects at the junctions between the re-entrant and strip line sections. In view of the large plate spacing compared to wavelength ( $b/\lambda = 0.405$  at 5.0 Gc in the re-entrant section), it is not surprising that discontinuity effects are severe. The poor directivity is also attributable to the discontinuities.

In order to decrease the coupling of the re-entrant section, the outer characteristic impedance,  $Z_{o1}$ , was reduced from 84.7 to about 76.3 ohms by reducing the plate spacing from 0.956 inch to 0.832 inch. This produced the desired triple loop appearance of the coupling and main line response curves. The center loop was 0.75 db, the lower loop 0.3 db, and the upper loop 0.7 db. The directivity was almost unaffected, however.

A number of changes were then made in the end sections to cancel discontinuity effects and increase the directivity. The final data for the 1-5-Gc hybrid coupler

is shown in Fig. 10. Photographs of the coupler disassembled and assembled are given in Figs. 11 and 12.

## CONCLUSION

The advantages of the re-entrant cross section have been verified experimentally in both single-section and three-section 3-db hybrid-coupler models. For cross-section heights in the range of 0 to  $0.16\lambda$ , the simple design formulas have proved to yield very good accuracy. Beyond  $0.16\lambda$  the coupling appears to increase gradually, but nevertheless good performance was achieved even with a cross section height as large as  $0.4\lambda$ , after a minor adjustment of dimensions.

In the case of a three-section design for 5:1 bandwidth, a center section coupling of  $-1.28$  db is required. This very strong coupling is easily achieved by the re-entrant cross section, and may be maintained accurately in production.

# Correspondence

## On the General Relation Between $\alpha$ and $Q$ \*

The relation between the quality factor  $Q$  and the attenuation constant  $\alpha$  of a transmission line has been known as follows:

$$\alpha = \frac{\beta}{2Q}$$

where  $\beta$  is the phase constant. Recently from the following relation of propagation constant at resonance

$$\Gamma(\omega_0) + \frac{\partial \Gamma}{\partial \omega} \Delta \omega \simeq i\beta(\omega_0),$$

where

$$\Gamma(\omega_0) = \alpha(\omega_0) + i\beta(\omega_0).$$

Yeh<sup>1</sup> derived a general relation between  $Q$  and  $\alpha$ , namely,

$$\alpha = \frac{v_p}{v_g} \frac{\beta}{2Q},$$

where  $v_p$ , and  $v_g$  are the phase velocity and group velocity of the wave respectively. This general relation can be derived very simply from the generally accepted definition of  $\alpha$  and  $Q$ .

General definition of  $Q$  applicable to waveguide as well as to ordinary transmission line is as follows:

$$Q = \omega \frac{\text{energy stored per unit length}}{\text{power lost per unit length}}.$$

The attenuation factor  $\alpha$  in the range of propagation is given by

$$\alpha = \frac{1}{2} \frac{\text{power lost per unit length}}{\text{power transmitted}}.$$

If we realize that the power transmitted is equal to the energy stored per unit length multiplied by  $v_g$  (group velocity) instead of  $v_p$  (phase velocity) then it is readily seen that

$$\alpha = \frac{1}{2} \frac{\omega}{v_g Q}.$$

Then from the relation

$$v_p = \frac{\omega}{\beta},$$

we obtain

$$\alpha = \frac{v_p}{v_g} \frac{\beta}{2Q}.$$

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## A Varactor Frequency-Modulated AFC Reference Cavity\*

### SUMMARY

A varactor frequency-modulated microwave cavity is described where the application of a periodic square wave voltage to a varactor serves to electronically detune the cavity. There results a discriminator characteristic which makes the device applicable as an AFC reference. Experimental results are given and discussed.

### INTRODUCTION

When an AFC loop is used to stabilize the frequency of a microwave signal source, a stable frequency reference is needed. A frequency modulated (periodically detuned) microwave cavity has been developed which performs this function by giving an AC error voltage whose magnitude and phase are proportional respectively to the magnitude and direction of the frequency shift of the source from the reference frequency of the cavity. The cavity thus has a discriminator characteristic. The periodic detuning of the cavity is accomplished through the application of a square wave voltage to a nonlinear reactance device (a varactor<sup>1</sup> or  $pn$ -junction diode) coupled to the cavity.

The varactor frequency-modulated microwave reference cavity is basically a high- $Q$

\* Received February 26, 1963.

<sup>1</sup> A. Uhlir, Jr., "The potential of semiconductor diodes in high frequency communications," *Proc. IRE*, vol. 46, pp. 1099-1115; June, 1958.

\* Received February 26, 1963.

<sup>1</sup> C. Yeh, "A relation between  $\alpha$  and  $Q$ ," *Proc. IRE (Correspondence)*, vol. 50, p. 2145; October, 1962.